Consequently,\[\lambda(A(v_x)) = \max_{x \in D} (A(v_x)x, x) = \frac{t}{\alpha_1 + \alpha_2 + \ldots + \alpha_s}.\]

From (7) and (8) we get that \(\lambda(m(v_x)) = \lambda(A(v_x)) = \nu\), i.e., \((\mu_0, \nu_0)\) are optimal strategies, \(\nu\) is the value of the game.

LITERATURE CITED


VARIOUS SCHEMES OF CONTINUOUS INSPECTION

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We consider various schemes of continuous inspection. We show their advantages and deficiencies.

Suppose it is necessary to organize inspection of a continuous flow of objects \(O_t\), \(t = 0, 1, 2, \ldots\), where each object is with probability \(q\) defective, with probability \(p = 1 - q\) nondefective. Two forms of inspection are possible here: under the first all defective objects discovered are corrected or replaced by nondefective ones, under the second they are removed without replacement. These forms of inspection are denoted, respectively, by \(CQC-I\) and \(CQC-2\) (cf., e.g., [1]).

As a rule in inspecting objects one checks them one element at a time [1; 2]. However, for example in testing objects for reliability when the time of testing one object is commensurable with the time released in testing a group of objects, inspection cannot be organized in this way. Moreover, application of procedures for grouped checks (if this is technically possible), which lower the cost of inspection also dictate inspecting by groups [3].

We describe the following general scheme for continuous inspection of objects.

1. The total inspection of objects is carried out in groups of \(n_1\) objects up to the appearance of \(s\) nondefective groups in succession. A group of objects is considered nondefective if the number of defective objects in the group is less than or equal to the previously given \(c_1\). After the appearance of a series of \(s\) nondefective groups one passes to sampled inspection.

2. Sampled inspection is done in groups of \(n_2\) objects up to the discovery of \(r\) defective groups after which one again passes to total inspection, a group being considered defective if the number of defective objects in it is greater than a previously given \(c_2\). Each group is inspected with probability \(x\).

By an inspection cycle we mean the process of inspection from the beginning of total inspection to the beginning of the next total inspection.

Clearly the general scheme of continuous inspection described above describes various inspection plans. Thus, for example, for \(n_1, n_2 = 1\) and \(c_1, c_2 = 0\) this is Dodge's plan of continuous inspection [1, 2].

Following [1], we write down the important characteristics of the general scheme of inspection. As a preliminary, we note that the probability of nondefectiveness of one group under total and sampled inspection, respectively, is equal to:

The expectation of the number of inspected groups up to the first appearance of a series of \( \ell \) nondefective groups in succession is equal to \((1-P_1^\ell)[P_1^\ell(1-P_1)]^{-1}\) [1]. The expectation of the number of inspected groups up to the appearance of \( r \) defective groups under sampled inspection is equal to \( r(1-P_2)^{-1} \) and the number of skipped ones is \( r(1-P_2)^{-1}(\infty^{-1}-1) \). Thus the following theorem is true.

**Theorem 1.** The expectation of the proportion of inspected objects in one inspection cycle can be calculated from the formula

\[
f(q) = \frac{\alpha}{\alpha + (1-\alpha)H},
\]

where

\[
H = \frac{n_2 r P_1^\ell(1-P_1)}{n_1(1-P_1^\ell)(1-P_1) + n_2 r P_1^\ell(1-P_1)}.
\]

**Proof.** The expectation of the proportion of inspected objects in one inspection cycle is equal to the ratio of the expectations of the number of checked objects and the number of accepted objects in an inspection cycle, which can be calculated, respectively, from the formulas

\[
M(\xi) = n_1 \frac{1-P_1^\ell}{P_1^\ell(1-P_1)} + n_2 \frac{r}{1-P_2} = \frac{n_1(1-P_1^\ell)(1-P_2) + n_2 r P_1^\ell(1-P_1)}{P_1^\ell(1-P_2)(1-P_2)},
\]

\[
M(\eta) = M(\xi) + n_2 r(\infty^{-1}-1) = \frac{n_1(1-P_1^\ell)(1-P_2) + n_2 r P_1^\ell(1-P_1) + n_2 r \infty^{-1} P_1^\ell(1-P_1)}{P_1^\ell(1-P_2)(1-P_2)},
\]

where

\[
(\infty^{-1}-1) = \sum_{i=1}^{\infty} (1-\alpha)^i \alpha.
\]

is the expectation of the skipped groups between two inspected groups under sampled inspection.

We find the ratio of (2) to (3) and we get

\[
f(q) = \frac{M(\xi)}{M(\eta)} = \frac{\alpha}{\alpha + (1-\alpha)H},
\]

where

\[
H = \frac{n_2 r P_1^\ell(1-P_1)}{n_1(1-P_1^\ell)(1-P_1) + n_2 r P_1^\ell(1-P_1)}.
\]

which is what had to be proved.

We note that the average level of output quality (AQQ) for the inspection plans CQC-1 and CQC-2 (cf. [1]) can be found from the formulas

\[
L_q(\xi) = q \left[ 1 - f(q) \right],
\]

\[
L_q(\eta) = \frac{q}{1 - q f(q)}.
\]

Substituting the value of \( f(q) \) into (4) and (5), we get that in our case Theorem 2 holds.

**Theorem 2.** The AQQ for inspection plans CQC-\( j \) (\( j = 1, 2 \)) is given by the formula

\[
L_q(\xi) = \frac{q(1-\alpha)P_1^{\ell-1} H}{\alpha + (1-\alpha)P_1^{\ell-1} H}.
\]

The Dodge inspection plan which is a special case of the above-described scheme of continuous inspection, is aimed at the detection of production errors, i.e., the discovery of an increase of the parameter \( q \).

In mass production there sometimes arises the problem of lowering the level of defectiveness of a collection of objects, in other words, the problem of extracting defective objects from the flow of production and lowering the parameter \( q \) with respect to its known initial value. When such a problem arises one can apply a scheme which it is convenient to