Let us denote by $M^e(N)$ the class of $n$-dimensional, centrally symmetrical polyhedra described around a sphere of
diameter $1 + e$ and having $2N$ faces; $M(N)$ will be denoted by $M(N)$.

We call the set $X \in \mathbb{R}^n$ a universal covering if for some motion any subset in $\mathbb{R}^n$ of unit diameter can be covered by
the image of $X$. Universal coverings that can be divided into three and four parts of diameter less than 1 were successfully
constructed in dimensions 2 and 3, respectively. This made it possible to prove in these dimensions Borsuk's conjecture on
the possibility of dividing any bounded subset in $\mathbb{R}^n$ into $n + 1$ parts of smaller diameter [1-4]. In [5] it was proved that for
$N > \frac{n(n+1)}{2}$ all the elements of $M(N)$ are not universal coverings in $\mathbb{R}^n$.

**THEOREM 1.** For $N < (\cos e)^{1-n}$ all the elements of $M^e(N)$ are universal coverings.

By a supporting function of a subset in $\mathbb{R}^n$ with respect to some point we mean the restriction of the usual supporting
function to a unit sphere $S^{n-1}$ with center at this point.

**LEMMA 1.** A supporting function $H$ of a body $K$ of constant width 1 in $\mathbb{R}^n$ with respect to the center of its
circumsphere of least radius satisfies the Lipschitz condition with constant $\sqrt{\frac{r^2 - \frac{1}{4}}{\frac{n-1}{4(n+1)}}}$.

Indeed, for $e \in S^{n-1}$ we have $\min(H(e), H(-e)) > \frac{1}{2}$, since $H(e) + H(-e) = 1$, and we can assume that
$H(e) > \frac{1}{2}$. Let $O$ be the center of the circumsphere of $K$, $A$ a point of $K$ in the supporting plane corresponding to vector
e, and let $\varphi(e)$ be the angle between $OA$ and $e$. Then $OA \cdot \cos \varphi(e) > \frac{1}{2}$ and for $e_i \in S^{n-1}$, which forms with $e$ angle $\varphi_i$,
we have $H(e_i) = OA \cdot \cos \varphi(e_i) + \varphi_i = OA \cos \varphi(e) - 2 \cdot OA \sin \frac{\varphi_i}{2} \sin \left(\varphi(e) + \frac{\varphi_i}{2}\right) \geq H(e) - \varphi_1 \cdot OA \sin \left(\varphi_1 + \frac{\varphi_2}{2}\right)$
whence follows estimate $\max_{e \in S^{n-1}} OA \sin \varphi(e)$ for the desired Lipschitz constant. But $OA \sin \varphi(e) = OA \sin$
$\sqrt{1 - \cos^2 \varphi(e)} \leq \sqrt{r^2 - \frac{1}{4}}$; this inequality follows from Young's theorem.

**LEMMA 2.** Let $A \subseteq S^{n-1}$ be measurable and let $\mu(A) > N^{-1} \mu(S^{n-1})$; then any $N$ points of sphere $S^{n-1}$
can be placed in $A$ by rotating the sphere.

Let $\mu_e$ be the normalized Haar measure on a group of rotations $SO(n)$. Set $A_i = \{a \in SO(n) | \text{ under rotation a}
the image of the i-th point does not occur in } A\}$. Then $\mu_i(A_i) = 1 - \mu(A)/\mu(S^{n-1}) < \frac{1}{N}$ and to prove Lemma 2 it is enough
to take any element of set $SO(n) \setminus \bigcup_{i=1}^{N} A_i$ of positive measure.

**Proof of Theorem 1.** Suppose that polyhedron $m \in M^e(N)$ and that $e_1, \ldots, e_N \in S^{n-1}$ are the directing vectors of
$N$ lines which are perpendicular to the pairs of parallel faces of $m$ with common origin at the center of the circumsphere of
body $K$ of constant width.

Let us consider the set $A = \{e \in S^{n-1} | H(e) = \frac{1}{2}\}$. This set is a set symmetric with respect to the center of the
sphere and dividing $S^{n-1}$ into two centrally-symmetric parts. We denote by $A_e$ a neighborhood of $A$ of radius $e > 0$ and by
$\mu$ the standard area measure on $S^{n-1}$. By isoperimetric considerations [6], $\mu(S^{n-1} \setminus A_e)$ is not greater than twice the measure
of a spherical cap of diameter which itself is less than the area of a hemisphere of diameter $\alpha = (\cos e)^{1-n}\mu(S^{n-1})$
According to the hypothesis of Theorem 1, $\mu_A > 1 - (\cos e)^{1-n} > 1 - \frac{1}{N}$. Therefore, according to Lemma 2, vectors
$e_1, \ldots, e_N$, and with them also polyhedra $m$ can be simultaneously rotated so that they all occur in $A_e$, but then, according
to Lemma 1, $H(e_i) = \frac{1}{2} < \frac{e}{2}$ and the rotated polyhedron $m$ covers $K$.
**Remarks.** It is obvious that Theorem 1 with proof admits the following generalizations.

1. If \( N < \left( \cos \frac{\epsilon}{\sqrt{4r^2 - 1}} \right)^{1-n} \), then any polyhedron \( m \in M^e(N) \) is a covering for bodies of constant width with radius of circumsphere at most \( r \).

2. Let the measurable \( A \subset SO(n) \) and let \( N \leq \mu_1(A) \cdot (\cos \epsilon)^{-n} \). Then for a polyhedron \( m \in M^e(N) \) having a common center \( O \) with the circumsphere of an arbitrary body \( K \) of constant width there exists a rotation \( a \in A \) around \( O \) such that \( K \subset am \), that is, \( m \) is an \( A \)-covering, following the terminology of [7, p.73].

3. In the conditions of Remark 2, \( \mu_1(\{ a \in SO(n) | K \subset am \}) > 1 - N \times (\cos \epsilon)^{-n} \).

2. The following theorem of the type of Dvoretzky's theorem [8] on an almost spherical projection refines the estimate of the dimension of the ambient space in the special case of bodies of constant width. Let us denote by \( n(\epsilon) \) the minimal number of elements of an \( \epsilon \)-mesh symmetric with respect to the center on a sphere \( S^{n-1} \subset \mathbb{R}^n \) with angular metric.

**THEOREM 2.** For \( N > \min \left( \frac{1}{2} n \sqrt{\frac{2}{\log_2 \frac{e}{\epsilon}}} - 1 \right) \), any body of constant width in \( \mathbb{R}^N \) has an \( \epsilon \)-aspherical, \( n \)-dimensional, orthogonal projection.

Let us prove that the conclusion of Theorem 2 is valid if \( N > \frac{1}{2} n \sqrt{\frac{e}{1 + 2\epsilon}} \).

**LEMMA 3.** Any \( n \)-dimensional polyhedron \( m \in M(N) \) covers an orthogonal, \( n \)-dimensional projection of any \( N \)-dimensional body of constant width.

**Proof.** We fix the following decomposition into a direct sum of two orthogonal subspaces: \( R^N = R^n \oplus R^{N-n} \). Let \( \pi: R^n \oplus R^{N-n} \to R^n \) be a projection onto the first term. For \( m \in M(N) \) we consider the polyhedral set \( \pi^{-1}(m) \). Let us denote by \( P_i \) a fiber of unit width \( \frac{1}{2} \leq x_i \leq \frac{1}{2} \), where \( n + 1 \leq i \leq N \). The set \( M = \pi^{-1}(m) \cap P_{n+1} \cap \cdots \cap P_N \) is a polyhedron from \( M(2N-N) \); we will show that it is a universal covering in \( \mathbb{R}^N \).

If \( M \) is not a universal covering, then, according to Theorem 1 of [9], a continuous mapping \( F: SO(N) \to S^{N-n-1} \), occurs. Since by construction the values of \( F \) do not change as \( M \) rotates around \( m \), a continuous mapping \( F: SO(N) / SO(N-n) = V_n,N \to S^{N-n-1} \), occurs, which we denote by the same letter \( F \). By construction, \( F \) has the following property: \( F(\omega t) = - F(\omega) \), where \( t \) is the involution of the sign change of the vectors of frame \( v \in V_n,N \), which contradicts Lemma 1 of [10] stating that there exists no \( G \)-homomorphism of a \( k \)-connected space into a \( k \)-dimensional cell space on which finite group \( G \) acts freely. Lemma 3 is proved.

Let us consider an \( n \)-dimensional, orthogonal projection of a body \( K \subset R^N \) of constant width \( 1 \) that can be covered by a polyhedron \( m \in M(N) \), bounded by supporting planes to an \( n \)-dimensional unit sphere at the points of some \( \sqrt{\frac{e}{1 + 2\epsilon}} \) mesh of \( n \left( \sqrt{\frac{e}{1 + 2\epsilon}} \right) \) elements invariant with respect to the center.

Let us show that diameter \( m \leq 1 + \frac{\epsilon}{2 + \epsilon} \). Indeed, for every line \( l \) issuing from the center of the sphere there exists a point from the chosen mesh the angular distance from which to the point of the intersection of \( l \) with the sphere is at most \( \sqrt{\frac{e}{1 + 2\epsilon}} \). The tangent hyperplane at point a cuts on \( l \) (computing from the center) a segment of length at most

\[
\frac{1}{2} \sqrt{1 + \frac{\epsilon}{1 + 2\epsilon}} \leq \frac{1}{2} \left(1 + \frac{\epsilon}{2 + 2\epsilon}\right) \leq \frac{1}{2} \left(1 + \frac{\epsilon}{2 + \epsilon}\right),
\]

QED.