Since \( p^n(00) > 0, \ p < 1 \), and (7) holds, it remains to take
\[
p = p_3 = 1.
\]

This signifies that the iteration process in the limit leads to an asymptotically reliable Sheffer element. Q.E.D.

We formulate our main result by combining the results of Lemmas 5 and 6.

THEOREM 1. In order that an unreliable element \( E^\phi \) with two binary inputs and one binary output be a Sheffer element, it is necessary and sufficient that the following conditions be fulfilled:

1) \( \forall x (\phi (ax) = \alpha \implies p_\alpha (ax) \neq 1) \);
2) \( \exists \exists \exists \exists (p_\alpha (\bar{z}) = p_\alpha (\bar{\bar{z}}) = 1) \);
3) \( \exists \exists \exists \exists (p_\alpha (\bar{z}) = 1) \land \exists \exists (p_\alpha (\bar{\bar{z}}) = 1) \land p(\bar{z}, \bar{\bar{z}}) > 1) \);
4) \( \phi \equiv \bar{D}_\phi \).

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LITERATURE CITED


THE INTEGRAL REPRESENTATION OF VECTOR MEASURES
ON A COMPLETELY REGULAR SPACE

O. E. Tsitritskii

We consider the vector space \( C(X, E) \) of all bounded continuous functions from a completely regular space \( X \) into a Banach space \( E \). It is given a special locally convex topology \( \xi \). We prove the analog of the Riesz–Markov theorem for the \( \xi \)-continuous linear operators which map \( C(X, E) \) into a Banach space \( F \).

Let \( T \) be a compact separable space and \( \varphi \) a continuous linear functional on the vector space of all continuous functions on \( T \) under the topology of uniform convergence. The well-known Riesz–Markov theorem states that there is a representation

\[
\varphi (f) = \int_T f \, d\mu,
\]

where \( \mu \) is a bounded regular Borel measure on \( T \). In this paper we will generalize this result in several directions. We consider continuous linear functionals on the space \( C_0(x) \)


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of all continuous functions on a locally compact space $X$ which "vanish at infinity" and the space $C_b(X)$ of all continuous functions on a locally compact space $X$ which are bounded. R. Buck [1] introduced a special locally convex topology on $C_b(X)$ which is called "strict." This topology permitted the development of a satisfactory connection between measure theory and topology. The appearance of V. S. Varadarain's paper [2] let a series of authors [3-5] apply Buck's approach to $C_b(X)$ when $X$ is a completely regular space. On the other hand, N. Dinculeanu [6] obtained an analog to the Riesz–Markov theorem for linear operators acting on the space $K(X, E)$ (where $K(X, E)$ is the set of all continuous mappings with compact support from a locally compact space $X$ to a Banach space $E$) into a Banach space $F$. In order to do this it turned out that a sufficiently satisfactory representation could be obtained only for so-called majorized operators and not for all continuous linear mappings in the uniform topology on $K(X, E)$.

In this note we consider $C(X, E)$, the vector space of all continuous bounded mappings of a completely regular space $X$ into a real Banach space $E$. It is given a specially locally convex topology $\xi$. The goal of this paper is the following theorem.

**THEOREM 1.** Any linear operator $U$ mapping $C(X, E)$ into a real Banach space $F$ is continuous in the topology $\xi$ if and only if for each $f \in C(X, E)$ there is a representation

$$U(f) = \int_X f \, dm,$$

where $m$ is uniquely defined on $U$ and is a regular Borel measure on $X$ with values in the Banach space $L(E, F)$ of all continuous linear mappings of $E$ into $F$ having finite variation.

This will also establish a connection between $\xi$ and the earlier mentioned "strict" topology of $C(X, E)$.

1. Throughout the paper $X$ will be a completely regular topological space. A Borel set in $X$ is an element of the $\sigma$-algebra $\mathcal{B}(X)$ generated by the open subsets of $X$. A countably additive set function which is defined on $\mathcal{B}(X)$ and which has values in a real normed space $G$ (in $[0, +\infty]$) is said to be a (vector) Borel measure $m$ (a positive Borel measure $\mu$). A measure $m(u)$ is said to be regular if for any $A \subseteq \mathcal{B}(X)$ and any number $\varepsilon > 0$ there is a compact subset $K \subseteq A$ and an open subset $U \supseteq A$ such that for any $B \subseteq \mathcal{B}(X)$ with the property that $B \subseteq U \setminus K$ we have $\|m(B)\|_G < \varepsilon$ ($\|\mu(B)\| < \varepsilon$). The variation of the measure $m$ is the real-valued set function $|m|$ defined in the following manner for any $A \subseteq \mathcal{B}(X)$:

$$|m|(A) = \sup \sum_{i \in I} |m(A_i)|,$$

where the supremum is taken with respect to all finite collections $\{A_i, i \in I\}$ of disjoint sets of $\mathcal{B}(X)$, contained in $A$. We say that $m$ has finite variation if $|m|(A) < +\infty$ for any $A \subseteq \mathcal{B}(X)$. If $m$ is a Borel measure with finite variation then $|m|$ is a finite positive regular Borel measure. In the sequel we will use results on vector measures defined on the abstract sets investigated in [6, Chaps. 1, 2].

Let $\beta X$ be the Stone–Cech compactification of the space $X$. Let $\mathcal{B}(\beta X)$ be the $\sigma$-algebra of Borel sets in $\beta X$.

**LEMMA 1.** Let $m$ be a regular Borel measure on $X$ with values in $G$ having finite variation. For any $A \subseteq \mathcal{B}(\beta X)$ set $\bar{m}(A) = m(A \cap X)$. Then $\bar{m}$ is a regular Borel measure on $\beta X$ with finite variation and

$$|\bar{m}|(\beta X) = \sup \{ |m|(K): K \subseteq X, K \text{ is compact} \}. \quad (2)$$

**Proof.** Set $v(A) = |m|(A \cap X), A \subseteq \mathcal{B}(\beta X)$. Then $v$ is a finite positive regular Borel measure on $\beta X$ satisfying (2) according to Lemma 4.3a in [3] and $\|m(A)\|_G \leq v(A)$. $\bar{m}$ is a Borel measure on $\beta X$ and $m$ is regular in light of the regularity of $v$. Moreover, for any $A \subseteq \mathcal{B}(\beta X)$ we have $0 \leq |\bar{m}|(A) \leq v(A)$. From the latter inequality it follows that $|\bar{m}|$ satisfies (2).

**LEMMA 2.** Let $n$ be a regular Borel measure on $\beta X$ with values in $G$ and let $\mu$ be a finite positive regular Borel measure on $\beta X$ such that (2) holds for $\mu$ and

$$|n(A)|_G \leq \mu(A) \quad (A \subseteq \mathcal{B}(\beta X)). \quad (3)$$