The above inequality, together with (17), completes the proof of Theorem 2.

In conclusion, we note that for \( r = 0 \) the statement of Theorem 2 is contained in [6].

LITERATURE CITED


\[
\omega \left( 1/n, f \right) = o \left( A_n \left( f, x \right) \right) \quad n \to \infty.
\]

at the isolated points \( x \in \left[ -\pi, \pi \right] \). We restrict ourselves to the case of equally spaced points \( t_v \); i.e., we set \( t_v = \nu \pi/n = -\pi, -\pi + 1, \ldots, n - 1 \). Without loss of generality, we can assume that \( \phi = 0 \leq x < \pi/n \) and \( f(0) = 0 \), since the problem can be reduced to this case by introducing auxiliary functions.

**Theorem.** If \( t_v = \nu \pi/n, \nu = -n, -n + 1, \ldots, n - 1, 0 \leq x < \pi/n \), and \( f(0) = 0 \), then the deviation of \( \tilde{f}_n(x) \) from \( \tilde{S}_n^*(\tilde{t}, x) \) is given by

\[
\sum_{j=1}^{n-1} \frac{\cos \left( t_\nu - x \right)}{2 \sin \left( t_\nu - x \right)/2} + \frac{1}{2n} \sum_{j=1}^{n-1} \left[ f(t_j) - f(t_{j+1}) \right] dt + O\left( \omega(1/n, f) \right).
\]

There exist continuous functions for which each of the first two terms satisfies Eq. (1).

**Proof.** We write \( f_n(x) - S_n^*(\tilde{t}, x) \) in the form

\[
\sum_{j=1}^{n-1} \frac{\cos \left( t_\nu - x \right)}{2 \sin \left( t_\nu - x \right)/2} + \frac{1}{2n} \sum_{j=1}^{n-1} \left[ f(t_j) - f(t_{j+1}) \right] dt + O\left( \omega(1/n, f) \right).
\]

Since \( 0 \leq x < \pi/n \) and \( f(0) = 0 \), for \(-2\pi/n \leq t \leq x : 2\pi/n \) we have \( f(t) = O \left( \omega(1/n, f) \right) \). Therefore, bearing in mind the equality \( \cot u/2 = 2/u + O(1) \) and the estimate

\[
\int_{-\pi}^{\pi} \cot u/2 du = O(1), \quad \alpha, \beta > 0,
\]

we obtain

\[
I_1 = \frac{1}{2n} \sum_{-n}^{n} f(t) \cot (t - x)/2 dt = \frac{1}{2n} \sum_{-n}^{n} f(x + u) \cot u/2 du = O\left( \omega(1/n, f) \right),
\]

\[
I_2 = \frac{1}{2n} \sum_{-n}^{n} f(t) \cot (t - x)/2 dt = \frac{1}{2n} \sum_{-n}^{n} f(x + u) \cot u/2 du = O\left( \omega(1/n, f) \right),
\]

i.e.,

\[
I_1 - I_2 = O\left( \omega(1/n, f) \right).
\]

Furthermore,

\[
I_3 = \frac{1}{2n} \sum_{-n}^{n} \left[ \frac{\cos \left( t_\nu - x \right)/2 - \cos \left( (2n + 1) \left( t_\nu - x \right)/2 \right)}{\sin \left( t_\nu - x \right)/2} \right] =
\]

\[
= \frac{1}{2n} \sum_{-n}^{n} \left[ \frac{2 \sin \left( (n + 1) \left( t_\nu - x \right)/2 \right)}{\sin \left( t_\nu - x \right)/2} \right] = \omega \left( 1/n, f \right) \sin \left( t_\nu - x \right)/2 = O\left( \omega(1/n, f) \right).
\]

The expression

\[
\frac{1}{n} \sum_{j=-n}^{n-1} f(t_j) \sin n \left( t_\nu - x \right)
\]

is a linear combination of interpolating Fourier coefficients of \( f(x) \), and so we can write

\[
I_4 = \frac{1}{2n} \left[ \sum_{j=-n}^{n-1} f(t_j) \sin \left( t_\nu - x \right) \right] = O\left( \omega(1/n, f) \right).
\]

Bearing in mind that \( \cot (t_\nu - x)/2 = \tan x/2 = O(1/n) \).