Integral equations are used to solve the problem of the limiting equilibrium state of plates with a system of parallel blind (surface) cracks, which are under tensile forces and bending moments, on the assumption that plastic deformations develop along the entire depth in the neighborhood of the cracks. The effect of the geometric and physicomechanical parameters on opening of the edges of the cracks is investigated.

Let us consider (in the Cartesian rectangular system of coordinates X, Y, Z) an infinite plate that is weakened by a periodic system of blind (surface) cracks $|X| < a_0$, $Y = 2k\ell$, $k = 0, \pm 1, \pm 2, \ldots$, $(h-2d) \leq Z \leq h$ ($2h$ is the plate thickness and $2d$ is the crack depth). The plate is acted on by tensile forces $N^0_2$ and bending moments $M^0_2$, normal to the length of the cracks. We assume that the behavior of the material, the level of the load, and the size of the cracks are such that plastic deformations develop in a narrow band along the entire thickness of the plate in a neighborhood of the cracks. Beneath each of the cracks, i.e., at $|X| \leq a_0$, $-h \leq Z \leq (h-2d)$, the membrane states are constant and equal to the yield stress $\sigma_y$ of the material, while constant normal force $N$ and bending moment $M$ act in the plastic deformation zones ahead of the cracks, i.e., at $a_0 < |X| < a$ ($a = a_0 + \rho$, $\rho$ is the length of the plastic zone ahead of the crack). The quantities $\rho$, $N$, and $M$ are unknown.

Using an analog of the Leonov—Panasyuk—Dugdale model, we replaced the plastic zones with lines (segments of zero thickness) of the break in the elastic displacements and angles of rotation, at which the normal force $N$ and the bending moment $M$ should satisfy the plasticity condition $[2, 6]$

$$\frac{N}{2h\sigma_y} + \frac{|M|}{2N^0\sigma_y} = 1. \quad (1)$$

Within the framework of the model adopted the plastic deformations in the neighborhood of cracks of length $2a_0$ are taken into account by introducing and considering new through cracks of length $2a$, at whose edges the following conditions are satisfied:

$$N_2(X) = \begin{cases} -N^0_2 + \sigma_y(h-d), & |X| < a_0, \\ -N^0_2 + N, & a_0 < |X| < a, \end{cases}$$

$$M_2(X) = \begin{cases} -M^0_2 - 2\sigma_yd(h-d), & |X| < a_0, \\ -M^0_2 + M, & a_0 < |X| < a. \end{cases} \quad (2)$$

In the linear theory of plates the problem of determining the stress—strain state, causing tension and bending, is disconnected. In the case under consideration this problem is connected because the cracks are distributed asymmetrically about the middle of the surface and a plastic zone of unknown length exists.

The problem of determining the stress—strain state of a plate with a system of parallel through cracks of length $2a$, using the method proposed in $[3, 5]$, reduces to two singular integral equations

$$\frac{\mu}{4} \int_{-1}^{1} F_i(\xi) \left[ a_i cth \frac{\mu(\xi - x)}{\ell} \pm K_i [\mu(\xi - x)] \right] d\xi = f_i(x), \ |x| < 1 \quad (i = 1, 2), \quad (3)$$

whose solution should satisfy the condition...
Fig. 1

\[ \int_{-1}^1 F_i(\zeta) d\zeta = 0. \]  

(4)

In Eqs. (3)

\[ F_i = \frac{d}{dx} [v], \quad F_2 = \frac{d}{dx} [\theta_2], \quad a_1 = 1, \quad a_2 = \frac{3 + \nu}{1 + \nu}, \]

\[ f_1(x) = \frac{N_x(x)}{2Eh}, \quad f_2(x) = \frac{3M_x(x)}{2Eh^2}, \quad \mu = \frac{\pi^2}{l}, \]

\[ x = \frac{X}{a}, \quad K_1(s) = \omega_1(s), \quad K_2(s) = \frac{1 - \nu}{1 + \nu} \omega_1(s), \]

\[ \omega_1(s) = \left(1 - \frac{s}{sh s}\right) \cosh \frac{s}{2}, \quad s = \mu (\zeta - x), \]

and \( [v] \) and \( [\theta_2] \) are jumps in the displacement and the angle of rotation.

Upon solving Eqs. (3) by the method of mechanical quadratures \([1]\), we can easily obtain formulas for determining the stress-intensity factors \( K_N \) and \( K_M \) \([4]\) corresponding to the normal force and bending moment in the neighborhood of the tip of a crack of length 2\(a\). Since plastic zones exist ahead of real cracks, however, the forces and moments at the end of these zones are limited and, therefore, the stress-intensity factors of the forces and moments should become zero, i.e.,

\[ K_N = 0, \quad K_M = 0. \]  

(5)

Conditions (1) and (5) are used to determine the unknown force \( N \), the moment \( M \), and the length \( \rho \) of the plastic zone. The opening of cracks is determined by

\[ \delta(x, z) = [v(x)] + Z [\theta_2(x)]. \]  

(6)

Thus, when we integrate the solution of the singular integral equations (3), taking conditions (1) and (5) into account, and substitute the result into (6), we determine the opening of the crack at any point. We note that when we replace \( \delta \) by the critical crack opening \( \delta_k \) Eq. (6) goes over into a criterial equation, which establishes the relation between the applied local, the crack size, and the physicomechanical parameters of the plate under conditions of the limiting equilibrium state.

When a plate is weakened by only one crack, the integral equations (3) can be recast in a form that admits a closed solution, whereupon the formula for determining the crack opening has the form

\[ \delta(X, Z) = \frac{2}{\pi} \left[ (X - a_0) \Gamma(a, X, a_0) - (X + a_0) \Gamma(a, X, -a_0) \right] \times \]

\[ \times \left[ \frac{N - 2\sigma_y (h - d) D_0}{E} - Z \frac{M + 2\sigma_y (h - d)}{D (3 - 2\nu - 4\nu)} \right], \]  

(7)

where

\[ D_0 = 2Eh, \quad D = \frac{2Eh^3}{3(1 - \nu^2)}. \]