For complex linear control systems we give a method of construction of amplitude- and phase-frequency characteristics and a definition of stability that allow us to choose the set of frequencies according to a convenience of design. The method is based on the interpolation-dichotomous method and is convenient for realization on a computer, and allows us to exclude jumps of a phase-frequency characteristic and define stability with a high order of precision.

In the design of systems of automatic control (SAC) the frequency methods are widely used. Using amplitude-phase-frequency characteristics (APFC) a project engineer can obtain important quantitative indices, namely stability with respect to the amplitude and phase, the resonance frequency, cut frequency, and so on.

In general, it is not possible to obtain analytic expressions for phase-frequency characteristics (PFC) and amplitude-frequency (AFC) of a sufficiently complex SAC. Especially we refer to continuous-discrete and discrete SAC, in which transformations of continuous transmitting functions into discrete ones should be made additionally. Therefore for a construction of APFC numerical methods are applied, and a change of \( \omega \) from \( \omega_{\text{min}} \) to \( \omega_{\text{max}} \) with some step (constant of variable) and a calculation of values \( A(\omega) \) of AFC and \( \varphi(\omega) \) of PFC for each \( \omega_i \) are needed.

If we know the transmitting functions of links of SAC and their interrelation, it is not hard to organize a procedure of computing values \( W(\omega) \) of the frequency transmitting function (FTF) for frequency \( \omega \). Then \( A(\omega) \) (here and in what follows the expression is given in decibels) and the main value \( \varphi(\omega) \) of PFC can be evaluated from the formulas

\[
A(\omega) = 20 \cdot \log_2 \left( \frac{\sqrt{\text{Re} W(\omega)}}{\text{Im} W(\omega)} \right);
\]

\[
\tilde{\varphi}(\omega) = \arctan \left( \frac{\text{Im} W(\omega)}{\text{Re} W(\omega)} \right)
\]

with the help of complex arithmetic that is included into the software of a computer. These formulas hold for continuous as well as for discrete transmitting functions.

The problem appears in the definition of a complete value \( \varphi(\omega) \) of PFC and stability with respect to the amplitude \( A^*(\omega) \) and the phase \( \varphi^*(\omega) \). It is necessary to choose a step of increase of the frequency \( h_i = \omega_i - \omega_{i-1} \) that excludes jumps of PFC under a change of the frequency from \( \omega_{\text{min}} \) to \( \omega_{\text{max}} \), and, moreover, to find values of the frequency \( \omega \) for which \( A(\omega) = 0 \) and \( \varphi(\omega) = \pm n\pi \) (\( n \) is an integer) that correspond to stability with respect to the phase \( \varphi^*(\omega) \) and the amplitude \( A^*(\omega) \).

Below we set out a method of constructing APFC, which allows us to solve the difficulties.

We give a range of frequencies \([\omega_{\text{min}}, \omega_{\text{max}}]\) (for discrete systems we have \( \omega_{\text{max}} \leq \pi/T \), where \( T \) is the quantization period of the system), a number \( N \) of points for which we shall construct APFC, and maximally admitted increase \( \Delta \varphi_{\text{max}} \) of PFC (where \( \Delta \varphi_{\text{max}} \in \{0, \pi\} \)).

By the reason of convenience of design the range of frequencies \([\omega_{\text{min}}, \omega_{\text{max}}]\) can be decomposed into intervals, for instance, in constructions of graphs of logarithmic characteristics with a constant step with respect to the logarithm of the frequency.
The values of PFC will be represented as $\varphi(\omega^*) = \tilde{\varphi}(\omega^*) + 2\pi m(\omega^*)$, where $\varphi(\omega^*)$ is the complete value of PFC for the frequency $\omega^*$, $\tilde{\varphi}(\omega^*)$ is the main value of PFC, $m(\omega^*)$ is the (integer) number of complete revolutions which are made by the hodograph about the origin and with the change of frequency from 0 to $\omega^*$.

The frequency $\omega_{\text{min}}$ should be chosen such that the main value $\tilde{\varphi}(\omega_{\text{min}})$ and of PFC coincides with its complete value $\varphi(\omega_{\text{min}})$. To get this the inequality $|\varphi(\omega_{\text{min}})| < 2\pi$ should be satisfied. Otherwise it is necessary to consider the possibility of introducing the initial values $\varphi(\omega_{\text{min}})$ of PFC into the algorithm.

Taking into account what has been said, according to (1) and (2), we define $\varphi_1 = \varphi(\omega_{\text{min}})$ and $A_1 = A(\omega_{\text{min}})$. Each next step consists of a definition of APFC for a frequency $\omega_i$, using the known APFC for the frequency $\omega_{i-1}$ ($i = 2, 3, ..., N$) and analysis of the interval $[\omega_{i-1}, \omega_i]$. We calculate the value $A(\omega_i)$ of AFC from the formula (1). In order to define the complete value $\varphi(\omega_i)$ of PFC, we divide the interval $[\omega_{i-1}, \omega_i]$ into subintervals in such a way that on each of them the increase of PFC is not bigger than the given value $\Delta \varphi_{\text{max}}^*$. The limits of the current subinterval considered will be denoted by $\omega_{01}, \omega_{02}$. For assigning them, we use the information about the behavior of PFC on the previous subinterval in the form of coefficient $k$, which is equal to the quotient of the length of the previous subinterval and the absolute increase of PFC on it:

$$k = \frac{\Delta \varphi}{|\Delta \varphi|} = \frac{(\omega_{02} - \omega_{01})/(\varphi(\omega_{02}) - \varphi(\omega_{01}))}{\Delta \varphi_{\text{max}}^*}.$$

In the study of the first interval $[\omega_1, \omega_2]$ the behavior of PFC is unknown; therefore we set $k = 0$. We assign the limits of the first subinterval: $\omega_{01} = \omega_{i-1}; \omega_{02} = \omega_{i-1} + k\Delta \varphi_{\text{max}}^*$.

If $k = 0$ or $\omega_{02} > \omega_i$, we set $\omega_{02} = \omega_i$, i.e., we combine subintervals with the current interval.

The main value of PFC for the frequency $\omega_{02}$ is

$$\varphi(\omega_{02}) = \begin{cases} \pi/2, & \text{Re } W(j\omega_{02}) < 0, \\ \bar{\varphi}(\omega_{02}), & \text{Re } W(j\omega_{02}) \geq 0 \text{ and } \text{Im } W(j\omega_{02}) \geq 0, \\ \pi + \bar{\varphi}(\omega_{02}), & \text{Re } W(j\omega_{02}) \geq 0 \text{ and } \text{Im } W(j\omega_{02}) < 0, \\ 3\pi/2, & \text{Re } W(j\omega_{02}) = 0 \text{ and } \text{Im } W(j\omega_{02}) < 0, \\ \pi/2, & \text{Re } W(j\omega_{02}) = 0 \text{ and } \text{Im } W(j\omega_{02}) > 0, \\ 0, & \text{Re } W(j\omega_{02}) = 0 \text{ and } \text{Im } W(j\omega_{02}) = 0, \end{cases}$$

where $\bar{\varphi}(\omega_{02}) = \arctan(\text{Im } W(j\omega_{02})/\text{Re } W(j\omega_{02}))$.

The increase of the main value of PFC on the subinterval is equal to $\Delta \varphi = \tilde{\varphi}_{02} - \tilde{\varphi}_{01}$.

Using the known $m(\omega_{01})$ we define $m(\omega_{02})$:

$$m(\omega_{02}) = \begin{cases} m_{01}, & |\Delta \varphi| \leq \Delta \varphi_{\text{max}}^*, \\ m_{01} - 1, & \Delta \varphi > 2\pi - \Delta \varphi_{\text{max}}^*, \\ m_{01} + 1, & \Delta \varphi < \Delta \varphi_{\text{max}}^* - 2\pi. \end{cases}$$

If neither of the inequalities for $\Delta \varphi$ in (4) is satisfied, we divide the subinterval in two in the logarithmic scale and consider it. Otherwise the complete value of PFC has increased less than $\Delta \varphi_{\text{max}}^*$. Then $\varphi(\omega_{02})$ is defined by the value $\tilde{\varphi}(\omega_{02})$ and $m(\omega_{02})$. If $\omega_{02} = \omega_i$, then PFC for $\omega_i$ is defined as $\tilde{\varphi}_i = \tilde{\varphi}_{02}, m_i = m_{02}$. The coefficient $k$ retains its value.

If $\omega_{02} \neq \omega_i$, we define the coefficient $k$ and the limits of the next subinterval: $k = (\omega_{02} - \omega_{01})/|\varphi_{02} - \varphi_{01}|; \ \omega_{01} = \omega_{o2}; \ m_{01} = m_{02}, \ \varphi_{01} = \varphi_{02}, \ \omega_{02} = \omega_{01} + \Delta \varphi_{\text{max}}^*$ and analyze them as previously.

Thus, after a finite number of steps, we obtain the value of PFC for $\omega_i$. If $\Delta \varphi_{\text{max}}^*$ is chosen sufficiently small (of order of units of degrees), then the probability of jumps of PFC on the previous subintervals is excluded practically. Taking into account the behavior of PFC on the previous subinterval it makes the probability even smaller, and, moreover, reduces the number of operations that are necessary to compute $\varphi(\omega)$.

After the definition of the next pair of values $A(\omega_i)$ of AFC and $\varphi(\omega_i)$ of PFC for each $\omega_i$, except $\omega_1 = \omega_{\text{min}}$, we carry out a test of occurrence of two events:

1. AFC on the interval $[\omega_{i-1}, \omega_i]$ intersects the abscissa axis in the direction "from up to down." This situation is identified when the following inequalities are satisfied simultaneously: