Combining inequalities (27) and (29), we obtain the following rate of convergence bound:

\[ \| u - u_h \|_{W^2_2(\Omega)} \leq M \varepsilon (\| f \|_{L^2(\Omega)} + \| h \|_{L^2(\Omega)}). \]

LITERATURE CITED


ACCURACY OF THE "CROSS" DIFFERENCE SCHEME FOR THE SYSTEM OF ACOUSTIC EQUATIONS

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A "cross" difference scheme is constructed by the integro-interpolation method for the system of acoustic equations and an a priori bound is derived in some norm weaker than \( L^2 \). The bound is used to prove convergence of the solution of the difference scheme at a rate \( O(\tau^2 + h^2) \) to the solution of the original differential problem in the class \( W^{2,2}_2(Q_T) \) and at a rate \( O(\tau + h) \) to the solution in \( W^{1,2}_2(Q_T) \).

Construction and analysis of difference schemes for the system of acoustic equations has been often considered in specialized literature. In particular, the stability of various difference schemes was investigated in [2, 3, 9, 10]. Difference schemes on nonregular triangular grids were constructed in [1] for two-dimensional equations of acoustics and their solutions were shown to converge in the mean at a rate \( O(\tau + h) \) to the solution of the differential problem in the class \( C^0(Q_T) \).

The wave equation is known to be reducible to the system of acoustic equations and conversely. For instance, the analysis of the accuracy of difference schemes for the wave equation has proved that the solution of the difference problem converges in the mean at a rate \( O(\sqrt{\tau} + \sqrt{h}) \) to the generalized solution \( u \in W^{1,2}_\infty(Q_T) \) of the linear wave equation [7]. The a priori bound obtained in [5] using the exact difference scheme operators has been used to prove the conver-
gence of the approximate solution at a rate $O(r^2 + h^2)$ to the generalized solution $u \in W^2_2(Q_T)$ of the wave equation with boundary conditions of the first kind.

In this paper, we apply the integro-interpolation method to construct a "cross" difference scheme for the system of acoustic equations and obtain an a priori bound in some norm weaker than $L^2$. This a priori bound is used to prove convergence of the difference solution at a rate $O(r^2 + h^2)$ to the solution of the original differential problem $u, p \in W^2_2(Q_T)$ and at a rate $O(r + h)$ to $u, p \in W^1_2(Q_T)$.

1. Statement of the Problem

Consider the system of acoustic equations in the cylinder $Q_T = \{0 < x < 1, 0 < t \leq T\}$,

\[ \frac{\partial u}{\partial t} + a \frac{\partial p}{\partial x} = f_1(x, t), \quad (1) \]

\[ \frac{\partial p}{\partial t} + a \frac{\partial u}{\partial x} = f_2(x, t), \quad (x, t) \in Q, \quad (2) \]

with given initial conditions

\[ u(x, 0) = u_0(x), \quad p(x, 0) = p_0(x) \quad (3) \]

and two types of boundary conditions

\[ u(0, t) = \mu_1(t), \quad u(1, t) = \mu_2(t) \quad (4) \]

or

\[ u(0, t) = \mu_1(t), \quad p(1, t) = \mu_2(t). \quad (5) \]

In what follows, problem (1)-(4) is called problem (A) and problem (1)-(3), (5) is called problem (B).

The mixed boundary-value problem

\[ p(0, t) = \mu_1(t), \quad p(1, t) = \mu_2(t) \quad (6) \]

is reduced to problem (A) by changing the variables $u$ to $p$ and $p$ to $u$. Eliminating $p(x, t)$ from Eqs. (1), (2), we obtain the wave equation

\[ a^2 \frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + f(x, t). \quad (7) \]

The initial conditions for $u, p$ give the initial conditions for Eq. (7),

\[ u(x, 0) = u_0(x), \quad \frac{\partial u}{\partial t}(x, 0) = u_1(x) \equiv f_1(x, 0) - a \frac{\partial p}{\partial x}. \]

The boundary conditions (4) for problem (A) correspond to the first boundary-value problem for the wave equation. Conditions (6) correspond to the second boundary-value problem,

\[ \frac{\partial u}{\partial x}(0, t) = v_1(t), \quad \frac{\partial u}{\partial x}(1, t) = v_2(t), \]