Sufficient conditions are found for complete parabolic surfaces to be cylindrical in Euclidean space. A complete \((l-2)\)-parabolic surface is a cylinder with \((l-3)\)-dimensional generators if the order of flattening of the surface is \(l-2\) and the strong zero-index is constant. Bibliography: 11 titles.

A surface \(F^l\) in the Riemannian space \(R^n\) is called \(k\)-parabolic if its second quadratic form with respect to each normal at an arbitrary point \(Q\) has at least \(k\) zero coefficients when in diagonal form. Chern and Kuiper [1] have introduced the zero-index \(\mu(Q)\) of a point of the surface \(F^l\) in the Riemannian space \(R^n\). It is equal to the maximal dimension of a subspace \(L(Q)\) of the tangent space \(T_Q F^l\) such that \(A(Q, \nu)y = 0\) for \(y \in L(Q)\), where \(A(Q, \nu)\) is the matrix of coefficients of the second quadratic form with respect to an arbitrary normal \(\nu\) at the point \(Q\). The surface is called strongly \(k\)-parabolic if \(\mu(Q) \geq k\) for \(Q \in F^l\).

Strongly \(k\)-parabolic surfaces are \(k\)-parabolic, but the converse is not true.

**THEOREM 1.** A complete \(k\)-parabolic \(l\)-dimensional surface of nonnegative Ricci curvature in Euclidean space is a cylinder with \(k\)-dimensional generators.

For strongly \(k\)-parabolic surfaces of nonnegative sectional curvature an analogous theorem has been proved by P. Hartman [2].

**LEMMA 1 [3].** Suppose a surface \(F^l\) of class \(C^0\) in \(E^n\) is the metric product \(F^{l-k} \times R^k\), where \(F^{l-k}\) is a manifold with intrinsic metric and \(R^k\) is Euclidean space. If the image of one of the generators of \(R^k(Q), Q \in F^{l-k}\), on the surface \(F^l\) is a plane \(E^k \subset E^n\), then the surface \(F^l\) is a cylinder with a \(k\)-dimensional generator.

**LEMMA 2 [4].** Through each point of a complete \(k\)-parabolic surface in Euclidean space passes a \(k\)-dimensional plane belonging to the surface.

**PROOF OF THEOREM 1:** By Lemma 2 a plane \(E^k\) lies on the surface. By a theorem proved by Cheeger and Gromoll [5] the surface is the metric product \(F^{l-k} \times R^k\), where \(F^{l-k}\) is a Riemannian manifold and \(R^k\) is Euclidean space. Applying Lemma 1, we obtain the required assertion.

An analog of Theorem 1 for spherical space is

**THEOREM 2.** Let \(F^l\) be a compact \(k\)-parabolic surface in spherical space \(S^n\) whose Ricci curvature is at least \(l - 1\). If \(k > 0\), then \(k = l\), and the surface \(F^l\) is a completely geodesic great sphere \(S^l\).

**LEMMA 3 [6].** Let \(R^n\) be a complete Riemannian space whose Ricci curvature is at least \(n - 1\). If there is a closed geodesic of length \(2\pi\) in \(R^n\), then \(R^n\) is isometric to the unit sphere \(S^n\).

**LEMMA 4 [7].** Let \(F^l\) be a compact smooth surface in \(S^{l+p}\) isometric to the unit sphere \(S^l\). If \(F^l\) contains a great circle, then \(F^l\) is a great sphere.

**PROOF OF THEOREM 2:** Since the analog of Lemma 2 holds for spherical space, \(F^l\) contains a great circle of length \(2\pi\). By Lemma 3 \(F^l\) is isometric to the unit sphere \(S^l\). By Lemma 4 \(F^l\) is a great sphere \(S^l \subset S^n\).

For strongly \(k\)-parabolic surfaces the analogous theorem was proved in [8].

In a general Riemannian space there are no theorems analogous to Theorems 1 and 2. Let \(F^l\) be a surface in a Riemannian space \(R^{l+p}\). The surface is invariant with respect to the curvature operator if at each point \(Q \in F^l\) the equality \(\langle R(X,Y)Z, \nu \rangle = 0\), where \(X, Y, Z\) are in \(T_Q F^l\), \(\nu\) is an arbitrary
normal vector at the point \( Q \), and \( R \) is the curvature tensor of the ambient space. If we choose coordinates in \( R^{l+p} \) such that \( F^l \) is the coordinate surface of the variables \( x_1, \ldots, x_l \), and the coordinate surfaces of the variables \( x_{l+1}, \ldots, x_{l+p} \) are orthogonal to the surface \( F^l \), then the invariance condition reduces to the condition \( R_{ijk\alpha} = 0 \) (\( i, j, k = 1, \ldots, l; \ \alpha = l + 1, \ldots, p \)) along the surface \( F^l \).

**Theorem 3.** Let \( F^l \) be a compact \( k \)-parabolic surface, invariant with respect to the curvature operator, in a Riemannian space \( R^{l+p} \) whose two-dimensional sectional curvature is positive. If \( k > lp/(p+1) \), then \( F^l \) is a completely geodesic submanifold in \( R^{l+p} \).

In the proof we make use of

**Lemma 5** [9]. Let \( F^l \) be a surface, invariant with respect to the curvature operator, in a Riemannian space \( R^{l+p} \) for which \( \mu_0 = \min \mu(Q) > 0 \). Then in \( F^l \) through a point \( Q \) for which \( \mu(Q) = \mu_0 \) there passes a completely geodesic \( \mu_0 \)-dimensional surface of the ambient space.

**Lemma 6** [10]. Let \( F^l_1 \) and \( F^l_2 \) be compact completely geodesic surfaces in a Riemannian space of positive sectional curvature. If \( p < l \), the surfaces intersect.

**Proof of Theorem 3:** Let \( Q_0 \) be a point on the surface \( F^l \) for which \( \mu(Q) = \mu_0 \). At points \( Q \in F^l \) sufficiently close to \( Q_0 \) we have \( \mu(Q) = \mu_0 \). By the inequality \( \mu > l - (l - k)(p+1)/2 \) and the requirement on the order of parabolicity of the surface \( \mu_0 > l/2 \). By Lemma 5 through the point \( Q_0 \) passes a compact completely geodesic surface \( F^{\mu_0}(Q_0) \subset F^l \). Let \( Q \in F^l \) be a point near \( Q_0 \) for which \( \mu(Q) = \mu_0 \) and which does not lie on \( F^{\mu_0}(Q_0) \). Through this point there also passes a compact completely geodesic surface \( F^{\mu_0}(Q) \), which must not intersect \( F^{\mu_0}(Q_0) \). But on the other hand, by Lemma 6 they do intersect. Therefore \( \mu_0 = l \), and the surface \( F^l \) is a completely geodesic submanifold in the Riemannian space \( R^{l+p} \).

Let \( F^l \) be a complete regular surface in \( E^n \) for all points of which \( \mu = k = \text{const} \). Then through each point of the surface there passes a unique plane \( E^k(Q) \) along which the tangent space is stationary. Let us pass a plane \( E^{n-k} \) perpendicular to \( E^k(Q_0) \) through the point \( Q_0 \in F^l \). In a neighborhood of the point \( Q_0 \) the surface \( F^{l-k} = F^l \cap E^{n-k} \) will be regular with radius-vector \( \rho(u_1, \ldots, u_{l-k}) \). Let \( e_{l-k+1}, \ldots, e_l \) be an orthonormal basis of \( E^k(Q_0) \), and let \( S_q(Q) = (q = l - k + 1, \ldots, l) \) be vectors of the plane \( E^k(Q) \) such that their orthogonal projection on the plane \( E^k(Q_0) \) coincides with \( e_q \). Let \( v^{l-k+1}, \ldots, v^l \) be Cartesian coordinates in the plane \( E^k(Q) \) with respect to the basis \( S_q(Q) \). Then the radius-vector of the surface \( F^l \) in the infinite strip containing the generator of \( E^k(Q_0) \) has the form

\[
r = \rho(u) + \sum_{q=l-k+1}^l S_q(u)v^q;
\]

\[
\rho_i = \frac{\partial \rho}{\partial u_i} \quad (i = 1, \ldots, l-k); \quad N_\alpha (\alpha = 1, \ldots, n-l = p) \text{ is an orthogonal basis of normals to the surface } F^{l-k} \subset E^{n-k}. \text{ Since the vectors } e_q, \rho_i, \text{ and } N_\alpha \text{ form a basis of } E^n \text{ at points } Q \in F^{l-k} \text{ near } Q_0, \text{ it follows that }
\]

\[
S_q = e_q + \sum c_q^i \rho_i + \sum \mu_\alpha^q N_\alpha.
\]

Since \( \frac{\partial r}{\partial v^i} = S_q \), it follows from Weingarten's formulas that

\[
\frac{\partial S_q}{\partial u_i} = \sum_{s=1}^{l-k} \Gamma^s_{iq} r_s + \sum_{t=l-k+1}^l \Gamma^l_{iq} S_t + \sum_{\alpha=1}^{p} A^\alpha_{iq} \nu_\alpha,
\]

where \( \nu_\alpha \) are normals to the surface \( F^l \), and \( \Gamma^s_{iq} \) and \( \Gamma^l_{iq} \) are the Christoffel coefficients of the surface \( F^l \). From the definition of the zero-index \( \mu \) and the choice of the system of coordinates it follows that \( A^\alpha_{iq} = 0 \).

On the other hand we obtain from (2) that

\[
\frac{\partial S_q}{\partial u_i} = \frac{\partial c^q_i}{\partial u_i} \rho_s + \frac{\partial \mu^q_\alpha}{\partial u_i} N_\alpha + c^q_i \frac{\partial \rho_s}{\partial u_i} + \mu^q_\alpha \frac{\partial N_\alpha}{\partial u_i}.
\]