The paper analyzes the stress–strain state in a cylindrical shell with variable rigidity in two directions. The shell is analyzed in a geometrically nonlinear framework under different loads and boundary conditions. The proposed approach reduces the nonlinear system of partial differential equations to a sequence of linear systems of ordinary differential equations. The latter are solved by discrete orthogonalization.

The purpose of this study is to analyze the effect of geometrical nonlinearity on the stress–strain state of a cylindrical shell of variable thickness in two directions. Different loads with different fixing conditions at the ends are considered. Numerical analysis of the stress–strain state of plates and shells in a geometrically nonlinear framework was previously carried out in [1, 3, 5, 6, 7].

The approach applied in this paper to the stress state of flexible shells was previously used in [5, 6] for circular plates and conical shells.

We start with the following equations [8, 9]:

**equations of equilibrium**

\[ R \frac{\partial^2 T_1}{\partial x^2} + \frac{\partial S}{\partial p} = 0; \quad R \frac{\partial T_2}{\partial x} + R \frac{\partial S}{\partial x} + \frac{\partial H}{\partial x} + Q_2 = 0; \]

\[ \frac{R}{R} \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial x} - T_2 + Rq = 0; \quad Q_1 = \frac{\partial M_1}{\partial x} + \frac{1}{R} \frac{\partial H}{\partial x} - \left( \frac{T_1 - M_2}{R} \right) \theta_1 - S \theta_2; \quad \theta_2 = \frac{1}{R} \frac{\partial M_2}{\partial x} + \frac{\partial H}{\partial x} - T_2 \theta_2 - S \theta_1; \]

**strain expressions**

\[ e_1 = \frac{\partial U}{\partial x} + \frac{1}{2} \theta_1^2; \quad e_2 = \frac{1}{R} \frac{\partial V}{\partial p} + \frac{W}{R} + \frac{1}{2} \theta_2^2; \quad e_{12} = \frac{\partial V}{\partial x} + \frac{1}{R} \frac{\partial U}{\partial x} + \theta_2 \theta_1; \quad \theta_1 = -\frac{\partial W}{\partial x}; \quad \theta_2 = -\frac{1}{R} \left( \frac{\partial W}{\partial x} - V \right); \quad \kappa_1 = \frac{\partial \theta_1}{\partial x}; \]

\[ \kappa_2 = \frac{1}{R} \frac{\partial \theta_2}{\partial p} - \frac{\partial \theta_1}{\partial x}; \quad \kappa_{12} = \frac{\partial \theta_1}{\partial x}; \]

**elasticity relationships**

\[ T_1 = \frac{Eh}{1-\nu^2} (e_1 + \nu e_2); \quad T_2 = \frac{Eh}{1-\nu^2} (e_2 + \nu e_1); \quad S = \frac{Eh}{2(1+\nu)} e_{12}; \]
Here the coordinate surface of the shell is expressed in the orthogonal system of coordinates $\alpha, \beta; U, V, W$ are the component displacements; $T_1, T_2, S, Q_1, Q_2, M_1, M_2, H$ are the forces and moments; $\theta_1, \theta_2$ are the rotation angles; $\epsilon_1, \epsilon_2, \epsilon_{12}, \kappa_1, \kappa_2, \kappa_{12}$ are the shearing and bending strains; $E$ is the modulus of elasticity; $h$ is the thickness of the shell; $\nu$ is the Poisson ratio; $R$ is the radius of the shell; $q$ is the normal load component. Adding boundary conditions to Eqs. (1)-(3), we obtain a nonlinear boundary-value problem. As the solving functions we take $U, V, W, \theta, \hat{S} = S + \frac{2}{R} H, T_1, \hat{Q}_1 = Q_1 + \frac{1}{R} \frac{\partial H}{\partial \beta}, M_t$.

After appropriate transformations, we obtain a system of nonlinear partial differential equations that describe the stress-strain state of the shell:

\[
\frac{\partial U}{\partial \alpha} = e_1 - \frac{1}{2} \frac{\partial \theta_1}{\partial \beta}^2; \quad \frac{\partial V}{\partial \alpha} = \epsilon_{12} - \frac{1}{R} \frac{\partial U}{\partial \beta} - \theta_1 \frac{\partial \theta_2}{\partial \alpha}; \quad \frac{\partial W}{\partial \alpha} = - \theta_1;
\]

\[
\frac{\partial \theta_1}{\partial \alpha} = \kappa_1; \quad \frac{\partial S}{\partial \alpha} = - \frac{1}{R} \frac{\partial T_2}{\partial \beta} - \frac{1}{R} \frac{\partial H}{\partial \beta} - \frac{1}{R} Q_2; \quad \frac{\partial Q_1}{\partial \alpha} = - \frac{1}{R} \frac{\partial Q_2}{\partial \beta} + \frac{1}{R} T_2 - q; \quad \frac{\partial M_1}{\partial \alpha} = Q_1 - \frac{1}{R} \frac{\partial H}{\partial \beta} + \frac{1}{R} \frac{\partial T_1}{\partial \beta} \theta_1 + S \theta_1.
\]

The other unknowns are expressed in terms of the solving functions,

\[
T_2 = v T_1 + \frac{E h}{R} \left( \frac{\partial V}{\partial \beta} + W \right) + \frac{E h}{R} \theta_1^2; \quad Q_2 = \frac{1}{R} \frac{\partial M_2}{\partial \beta} + \frac{1}{R} \frac{\partial H}{\partial \beta} - T_2 \theta_2 - S \theta_1;
\]

\[
\theta_2 = - \frac{1}{R} \frac{\partial W}{\partial \beta} - V; \quad \epsilon_1 = \frac{1-v}{E h} T_1 - \frac{V}{R} \frac{\partial W}{\partial \beta} - \frac{V}{R} W - \frac{V}{R} \theta_2;
\]

\[
\epsilon_{12} = \frac{1}{E h} S; \quad \kappa_1 = \frac{1}{E h} \left( \frac{1}{1-\nu^2} M_1 - \frac{1}{R} \frac{\partial \theta_2}{\partial \beta} + \frac{1}{2R} \theta_1 \right);
\]

\[
M_2 = v M_1 + \frac{E h}{12R^2} \left( \frac{\partial W}{\partial \beta} + \frac{\partial V}{\partial \beta} - \frac{E h}{24R} \theta_1^2 \right) - \frac{E h}{12R^2 (1+\nu)} H = \frac{E h }{12R (1+\nu)} \frac{\partial \theta_1}{\partial \beta} + \frac{H}{12R^2 (1+\nu)} \theta_1 \left( \frac{\partial W}{\partial \beta} + V \right),
\]

where $\hat{S} = S + (2/R)H$.

Applying a difference approximation in the circular coordinate, we reduce the nonlinear boundary-value problem to a one-dimensional problem. The latter is solved by quasilinearization [2].

The original nonlinear boundary-value problem is thus reduced to a sequence of linear one-dimensional boundary-value problems which are solved by the stable numerical method of discrete orthogonalization [4]. As the starting approximation, we take the solution of the linear problem.

This approach was applied to study the behavior of a geometrical nonlinearity on the stress-strain state of a cylindrical shell with variable rigidity in two directions for various loads and boundary conditions.

1. Shell with Load of the Form $q = q_0 (1 + 0.3 \cos \beta) \sin(\alpha \pi / l)$

The problem was solved for $l = 100$ cm, $R = 100$ cm, $E = 6.5 \cdot 10^8$ kg/cm$^2$, $\nu = 0.3$, where $l$ is the length of the shell. The ends of the shell are hinged, $U = V = W = M_1 = 0$ at $l = 0$ and $l = 100$.

The following thicknesses and loads were used:

1) $q = q_0 (1 + 0.3 \cos \beta) \sin(\alpha \pi / l) \sin(\alpha \pi / l)$; $h = 0.5 (1 + 0.2 \cos \beta)$, where $q_0 = -10$;

2) $q = -20$; $-30$; 2) $q = -40 (1 + 0,3 \cos \beta) \sin(\alpha \pi / l) \sin(\alpha \pi / l)$; $h = 0.5 (1 + 0.15 \cos \beta)$;

3) $q = -40 (1 + 0.3 \cos \beta) \sin(\alpha \pi / l)$; $h = 0.5 (1 + 0.2 \cos \beta)$; 4) $q = -48 (1 + 0.3 \cos \beta) \sin(\alpha \pi / l)$; $h = 0.5 (1 + 0.15 \cos \beta)$, where $\gamma = -0.1$, $0.1, 0.2$. 

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