For an affine connection on the tangent bundle T(M) obtained by lifting an affine connection on M, the structure of vector fields on T(M) which generate local one-parameter groups of projective and affine collineations is described. On the T(M) of a complete irreducible Riemann manifold, every projective collineation is affine. On the T(M) of a projectively Euclidean space, every affine collineation preserves the fibration of T(M), and on the T(M) of a projectively non-Euclidean space which is maximally homogeneous (in the sense of affine collineations) there exist affine collineations permuting the fibers of T(M).

§1. Definitions and Statement of the Results. A vector field v on a manifold M with a symmetric affine connection $\Gamma$ is called a projective (affine) Killing vector field, if it generates a local one-parameter group of projective (affine) collineations. As is well known (see e.g. [1, p. 130]), projective Killing vectors are characterized by the condition

$$L_v \Gamma = \delta \otimes p + p \otimes \delta,$$

where $\delta$ is the unit affinor and $p$ is some covector. In the case of affine Killing vectors, the condition is

$$L_v \Gamma = 0.$$

Following Yano and Kobayashi [2], we introduce on the tangent bundle T(M) a symmetric affine connection $\bar{\Gamma}$, which we call the natural lift of $\Gamma$. If in T(M) we introduce the coordinates $z^\alpha = (\xi^\alpha, x^\alpha)$, where the $\xi^\alpha$ are local coordinates in M, and the $x^\alpha$ are the coordinates of the tangent vector $x$ of M at the point $(\xi^\alpha)$ with respect to the natural frame $\partial/\partial \xi^\alpha$, then the components of $\bar{\Gamma}$ can be written in the form

$$\bar{\Gamma}_{ab}^\nu = \begin{cases} 0, & \text{if } u(v) < 2, \\ \Gamma_{ab}^\nu, & \text{if } u(v) = 2, \\ x^q \partial_a r_{r}^\nu, & \text{if } u(v) = 3. \end{cases}$$

We are using here the method of indexing the components of objects on T(M) proposed by us earlier [3] in connection with introducing the above special coordinates.

Basic to all that follows is the tool of the complete lift from M to T(M) of tensors of arbitrary orders [3, 4]. In particular, every vector $\bar{v}$ on T(M) can be represented in the form

$$\bar{v} = [v, v],$$

which in the special coordinates signifies

$$\bar{v}^a = \begin{cases} v^a, & \text{if } v = 0, \\ 0, & \text{if } 0 < v < 1, \\ 1, & \text{if } v = 1. \end{cases}$$

where \( v, \dot{v} \) are vectors on \( M \), depending in general not only on the point \( \xi \in M \), but also on \( x \in T_\xi M \).

Let us define the methods of obtaining vectors on \( T(M) \) which are needed in the sequel:

1. vertical lift of a vector \( v \):
   \[
   V_v = r[0, v];
   \] (1.3)

2. horizontal lift of the vector \( v \):
   \[
   H_v = r[v, 0];
   \] (1.4)

3. natural lift of the vector \( v \):
   \[
   N_v = r[v, V_v], \quad V = V_x;
   \] (1.5)

4. vertical vector lift of the affinor \( f \):
   \[
   V_x f = r[f, x];
   \] (1.6)

5. horizontal vector lift of the affinor \( f \):
   \[
   H_x f = r[f, 0];
   \] (1.7)

Note that \( V_v \) and \( V_x f \) lie in the distribution of subspaces formed by the tangents to the fibers of \( T(M) \), and that \( H_v \) and \( H_x f \) lie in the distribution of horizontal subspaces defining the connection \( \Gamma \); \( N_v \) generates the local one-parameter group of transformations which are the differentials of the transformations of the group generated by \( v \). Also, \( H_x \delta \) is the geodesic spray for the connection \([5], [6, p. 160]\), and \( V_x \delta \) the Liouville vector field \([6, p. 158]\).

We have the following theorem about the canonical decomposition of a projective Killing vector on \( T(M) \).

**Theorem 1.** Let \( M \) be a differentiable manifold of dimension \( n \) (>2) with a symmetric affine connection \( \Gamma \). Then, in order that a vector \( \bar{v} \) on \( T(M) \) be projective Killing with respect to \( \Gamma \), it is necessary and sufficient that it be of the form

\[
\bar{v} = N u + V_v + H_x f + V_x g + p(x) V_x \delta,
\] (1.8)

where \( u \) is a projective Killing vector on \( M \):

\[
L_u \Gamma = \delta \otimes q + g \otimes \delta;
\] (1.9)

\( v \) is an affine Killing vector on \( M \):

\[
L_v \Gamma = 0;
\] (1.10)

\( p \) and \( q \) are covariantly constant covectors on \( M \), and \( f \) and \( g \) affinors on \( M \) satisfying the conditions

\[
\nabla f = \delta \otimes p, \quad R (f \cdot X, Y) = 0,
\] (1.11)

\[
\nabla g = -\delta \otimes q, \quad R (g \cdot X, Y) - g \circ R (X, Y) = 0
\] (1.12)

for all vectors \( X \) and \( Y \) on \( M \). And then \( L_\xi \Gamma = \tilde{\delta} \otimes \tilde{p} + \tilde{p} \otimes \tilde{\delta} \), where \( \tilde{p} = r[p, q] \).

In particular, for \( p = q = 0 \), this yields:

**Theorem 2.** In order that the vector \( \bar{v} \) on \( T(M) \) be affine Killing with respect to \( \Gamma \), it is necessary and sufficient that it be of the form

\[
\bar{v} = N u + V_v + H_x f + V_x g,
\] (1.13)

where \( u \) and \( v \) are affine Killing vectors of \( M \), and \( f \) and \( g \) are covariantly constant affinors on \( M \) satisfying the conditions

\[
R (f \cdot X, Y) = 0,
\] (1.14)

\[
R (g \cdot X, Y) - g \circ R (X, Y) = 0
\] (1.15)

for arbitrary vectors \( X \) and \( Y \) on \( M \).