i.e., one integrates with respect to \( y \) in the expressions for \( \tilde{k}_t(0) \) and \( \tilde{k}_q(0) \) easily, and the rest of the calculations are analogous to those given above.

LITERATURE CITED


SOLVABILITY OF THE BASIC INITIAL-BOUNDARY PROBLEM FOR THE EQUATIONS OF MOTION OF AN OLDROYD FLUID ON \((0, \infty)\) AND THE BEHAVIOR OF ITS SOLUTIONS AS \( t \to +\infty \)

A. A. Kotsiolis and A. P. Oskolkov

The solvability in the large on \((0, \infty)\) of the first initial-boundary problem for the equations of motion of an Oldroyd fluid with two spatial variables is proved and the connection as \( t \to \infty \) of the solutions of this problem with the solution of the analogous problem for the Navier--Stokes equation is investigated.

1. By an Oldroyd fluid (more precisely, an Oldroyd fluid of order \( L = 1 \)) is meant a linear viscoelastic fluid whose defining equation connecting the deviator of stress \( \sigma \) and the rate of deformation tensor \( D \), has the form \([1-2]\):

\[
\sigma + \lambda \frac{\partial \sigma}{\partial t} = 2\eta \partial \sigma + 2\eta \frac{\partial D}{\partial t}, \quad \lambda, \nu, \chi > 0, \quad \nu - \chi \lambda^{-1} > 0.
\]

(1)

It is shown in the papers of Oskolkov \([3-7]\) that the motion of an Oldroyd fluid can be described by the system of differential equations

\[
\begin{align*}
L(v) &= \frac{\partial \nu}{\partial t} + \nu \frac{\partial \sigma}{\partial x} - \mu \Delta v - \Delta u + \nu \Delta \Delta \nu + \nu \Delta u - \gamma \Delta \partial u + \nu \Delta \partial u + \gamma \Lambda \lambda^{-1} v, \\
\mu &= \chi \lambda^{-1}, \quad \sigma = \lambda(v - \mu)^{-1}, \quad \beta = (v - \mu)^{-1}.
\end{align*}
\]

(2)

The basic initial-boundary problem for (2) is the solution of (2) in \( Q_T = \Omega \times (0, T), \Omega \in \mathbb{R}^n, \nu = 2, 3, 0 < T < \infty \), subject to the initial-boundary conditions

\[
\begin{align*}
v(x, 0) &= v_0(x), \quad u(x, 0) = 0, \quad x \in \Omega; \quad v|_{\partial \Omega_T} = u|_{\partial \Omega_T} = 0.
\end{align*}
\]

(3)

The unique classical solvability of the initial-boundary problem (2)-(3) in the large \( \forall T < \infty \) is proved in the papers of Oskolkov \([3-7]\), if \( \Omega \) is a two-dimensional bounded domain, and in the small if \( \Omega \in \mathbb{R}^3 \). In the present paper we prove the unique classical solvability of the initial-boundary problem (2), (3) for \( \Omega \in \mathbb{R}^3 \) in the large on \((0, \infty)\), and we prove that under certain conditions on the data of the problem the solution of the problem (2), (3)
tends, as $t \to +\infty$, to a solution $v^*$ of the initial-boundary problem for the Navier–Stokes equation

$$
\frac{\partial v^*}{\partial t} + \nabla p^* + \nu \Delta v^* + \nabla \cdot \mathbf{u} = 0, \quad \mathbf{u} \cdot \nabla v^* = 0; \quad \frac{\partial p^*}{\partial t} = \nu \Delta v^*; \quad v^*|_{t=0} = v_0(x), \quad \frac{\partial v^*}{\partial n}|_{\partial Q} = 0. \tag{4}
$$

2. THEOREM 1. Let the following conditions hold: $\Omega$ is a two-dimensional bounded domain; $\partial \Omega \in C^2$; $v_0(x) \in C^2(\partial \Omega) \cap L^2(\Omega)$; $\mathbf{u}_0(x) \in L^2(\Omega)$; $\mathbf{u}_0 \in L^2(Q_T)$; $\mathbf{u}_0 \in L^2(\Omega_T)$; $\mathbf{u}_0 \in L^2(\Omega_T)$. Then the initial-boundary problem (2), (3) has a unique solution $(v, u, p)$ such that

$$
\begin{align*}
&v(x, t) \in W^{1,2}(\Omega_T; C^0(\partial \Omega); C^0(\partial \Omega)) \cap L^2(\Omega_T), \\
&u(x, t) \in W^{1,2}(\Omega_T; C^0(\partial \Omega); C^0(\partial \Omega)) \cap L^2(\Omega_T), \\
&\mathbf{u}_0 \in L^2(\Omega_T; C^0(\partial \Omega)), \\
&\mathbf{u}_0 \in L^2(\Omega_T, C^0(\partial \Omega)), \\
&p(x, t) \in L^2(\Omega_T; C^0(\partial \Omega)), \\
&\mathbf{u}_0 \in L^2(\Omega_T; C^0(\partial \Omega)),
\end{align*}
$$

and for it one has the estimate

$$
\begin{align*}
\|v\|_{W^{1,2}(\Omega_T)} + \|u\|_{W^{1,2}(\Omega_T; C^0(\partial \Omega))} &+ \frac{\partial u}{\partial n}\bigg|_{\partial \Omega_T} + \frac{\partial \mathbf{u}}{\partial n}\bigg|_{\partial \Omega_T} + \frac{\partial \mathbf{u}}{\partial n}\bigg|_{\partial \Omega_T} + \\
&+ \|p\|_{L^2(\Omega_T; C^0(\partial \Omega))} \leq \mu(t) \|v_0\|_{L^2(\Omega_T; C^0(\partial \Omega))},
\end{align*}
$$

where the constant $\mu(t)$ does not depend explicitly on $T$, $0 < T < \infty$.

As in the proof of the solvability of the initial-boundary problem for the two-dimensional Navier–Stokes system [8], to prove Theorem 1 it suffices to get the following estimates for solutions of the problem (2), (3), uniformly with respect to $0 < T < \infty$:

$$
\frac{1}{\nu} \max_{[0, T]} \left( \|v\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 + \|u\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 \right) \leq A T, \tag{6}
$$

$$
\max_{[0, T]} \left( \|v\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 + \|u\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 \right) \leq \left( \|v_0\|_{L^2(\Omega_T; C^0(\partial \Omega))} + \|f\|_{L^2(\Omega_T; C^0(\partial \Omega))} \right) \exp \left( A T^2 \right). \tag{7}
$$

$$
\frac{1}{\nu} \|v\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 + \frac{\nu}{2} \|u\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 \leq \left( \|v_0\|_{L^2(\Omega_T; C^0(\partial \Omega))} + \|f\|_{L^2(\Omega_T; C^0(\partial \Omega))} \right) \exp \left( A T^2 \right). \tag{8}
$$

With the help of (6)-(8) one can prove, by the method of Galerkin (cf. [8, 3-6]), that the problem (2), (3) has a unique generalized solution $(v, u)$ in the sense of Ladyzhenskaya [8, 3-6], and after this, with the help of the imbedding theorems of S. L. Sobolev and the theorems of the classical solvability of the corresponding linearized problem [3-6] one proves Theorem 1 in its full extent.

One gets (6)-(8) from the equations

$$
\frac{1}{\nu} \frac{d}{dt} \left( \|v\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 + \|u\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 \right) + \mu \|v\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 + \frac{\partial u}{\partial n}\bigg|_{\partial \Omega_T} = 0, \tag{9}
$$

$$
\frac{1}{\nu} \frac{d}{dt} \left( \|v\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 + \|u\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 \right) + \frac{\partial u}{\partial n}\bigg|_{\partial \Omega_T} + \int_\Omega v \partial_x v \, dx - \int_\Omega v \partial_x v \, dx = 0, \tag{10}
$$

which, in turn, one gets, respectively, from the equations

$$
\int_\Omega \partial_t v \cdot v \, dx = \int_\Omega v \partial_t v \, dx + \int_\Omega \partial_x \partial_x v \cdot v \, dx, \quad 0 < t < T. \tag{11}
$$

In fact, from (9) we have first of all, the inequality:

$$
\frac{1}{\nu} \frac{d}{dt} \left( \|v\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 + \|u\|_{L^2(\Omega_T; C^0(\partial \Omega))}^2 \right) \leq A T, \tag{12}
$$

from which the next inequality follows: