trices of the generators $N^a$ and $N^\alpha$, and an analysis of the structure of the basic matrices $\gamma^0$, $\gamma^\alpha$, and $\Lambda$ shows that they all are expressed in terms of the corresponding $\gamma^0$ and $\gamma^\alpha$ of the matrix of free relativistic first-order wave equations.

The authors are grateful to L. F. Babichev for a discussion of the results.

LITERATURE CITED


GROUP-THEORETIC DESCRIPTION OF EXTERNAL GAUGE FIELDS

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UDC 519.46:539.12

For an arbitrary external gauge field we construct an infinite group $\Gamma$ which contains all the information about the given field and describes some of its properties. We construct a field representation of the group $\Gamma$. We show that covariant derivatives become translation generators in such a representation of the group $\Gamma$. This allows us to interpret transformations from the group $\Gamma$ as motions in an external gauge field.

The basic work by Wigner [1] revealed a way to construct a theory of free elementary particles on the basis of representations of the Poincaré group. This led to the proof of the CPT-theorem and the theorem about the connection between spin and statistics, and to the derivation of the equations of motion for free elementary particles, etc. [2-4]. A number of approaches have been proposed to accomplish an analogous group-theoretic description of particles in external gauge fields [5]. These approaches, however, do not themselves contain the information about the gauge field; it is introduced only in the construction of the field representation.

In this work we demonstrate that a unique "dynamical" unification of the internal and external symmetry is possible on the basis of an infinite group $\Gamma$, described below. This group contains all the information about the external gauge field. The group $\Gamma$ is characterized by structure functions (analogous to the structure constants in the theory of Lie groups), which can be identified with the stress tensor of the external gauge field described by the group $\Gamma$. We construct a field representation of the group $\Gamma$, the carrier space of which is identified with the space of localized particle states in an external gauge field. We show that covariant derivatives generate translations in such a representation of the group $\Gamma$. This allows us to interpret transformations from the group $\Gamma$ as motions in an external gauge field.

1. Let $G = P \otimes V$, where $P$ is the proper orthochronous Poincaré group describing space-time symmetry (excluding reflections) and $V$ is the $n$-parameter Lie group of internal sym-
In canonical coordinates, elements of the group P are parametrized by the translation 4-vectors $t^\mu$ and by the parameters $\omega^{\rho\sigma}$ of the Lorentz transformations $\Lambda^\nu_\mu(\omega) = \exp(\omega)^\nu_\mu$. The indices $\mu, \nu, \rho, \sigma$ take on the values 0, 1, 2, and 3. The indices $i, j, k$ number the canonical coordinates $v^i$ of the elements from the group $V$. An arbitrary element $g$ of the group $G$ is given by the triplet

$$g = (v, \Lambda(\omega), t).$$

We label the coordinates of elements of the group $G$ by indices $\alpha, \beta, \gamma$ which take on values from 1 to $n + 10$ in such a way that the $g^\alpha$ are coordinates of: the element $v$ when $\alpha = 1, \ldots, n$; the Lorentz group when $\alpha = n + 1, \ldots, n + 6$; and the translations corresponding to the element $g$ in the sense of Eq. (1) when $\alpha = n + 7, \ldots, n + 10$. Therefore, we shall write: $\alpha = i$ for $\alpha = 1, \ldots, n$; $\alpha = (\mu\nu)$ for $\alpha = n + 1, \ldots, n + 6$; and $\alpha = \mu$ for $\alpha = n + 7, \ldots, n + 10$. The multiplication law for the elements of the group $G$: $(g_1, g_2) \rightarrow g_3$ is written in the form

$$g_3 = \gamma(g_1, g_2),$$

where the function $\gamma(g_1, g_2)$ is determined by the equalities

$$\gamma(g_1, g_2) = \gamma^a(v_1, v_2), \quad \gamma^a(g_1, g_2) = t^a_1 + t^a_2, \quad \gamma^{(\mu\nu)}(g_1, g_2) = \omega^{(\mu\nu)},$$

Here, $\gamma^a(\omega_1, \omega_2)$ gives the multiplication law in the group $V$ and $\omega^{(\mu\nu)}$ are determined by the conditions $\Lambda^\nu_\mu(\omega_1) = \Lambda^\nu_\mu(\omega_1)\Lambda^\nu_\mu(\omega_2)$. In these equalities $v_1$, $\omega_1$, and $t_1$ correspond to the element $g_1$; and $v_2$, $\omega_2$, and $t_2$ to the element $g_2$ in the sense of Eq. (1). Structure constants of the group $G$ are defined in the usual way [6]:

$$F^{(g)}_{\alpha\beta} = \left( \frac{\partial^2}{\partial g^\alpha g^\beta} - \frac{\partial^2}{\partial g^\beta g^\alpha} \right) \gamma^a(g_1, g_2) |_{g^a}.$$ (3)

Among them, the only nonzero constants are $F^{(g)}_{\alpha_1\alpha_2}$, $F^{(g)}_{\alpha_1(\mu\nu)}$, pertaining to the Poincaré group, and the structure constants of the group $V$.

2. We consider the set of smooth mappings of the Minkowski space $M$ into $G$, given in terms of coordinates in $M$ and $G$ by the functions $a(x)$, which will be written as a triplet

$$a(x) = (v(x), \Lambda(\omega(x)), t(x)).$$

Among these mappings, we separate out the subset $\Omega$ for which the following condition is satisfied:

$$\det\left( \partial_i \Lambda^\nu_\mu(\omega(x)) + t^\nu_i(x) \right) > 0 \quad \forall x \in M.$$ (5)

Using the multiplication law in the group $G$ (given by Eq. (2)) we define the multiplication operation in $\Omega$: with arbitrary $a_1$ and $a_2 \in \Omega$ we associate the mapping $a_3 \in \Omega$, given by the function

$$a_3(x) = \gamma(a_1(x), a_2(x)), \quad a_3(x) = (x^a(x), \Lambda(\omega(x)) + t(x)),$$

where the mappings $\omega_i(x)$ and $t_i(x)$ correspond to $a_i(x)$ in the sense of Eq. (4). It is easy to verify that the multiplication law defined in this way satisfies all properties of the group multiplication law. The unit element is given by the function $\gamma(x) = 0 \forall x \in M$. The existence of inverse elements is guaranteed by the condition (5). Therefore, the set $\Omega$ with the multiplication law introduced becomes a group, which we shall denote by $\Gamma$. We note that the group $\Gamma$ differs from the gauge group $G(x)$ with the structure $\Lambda(x)$, usually defined in $C(M, G)$. The gauge group of internal symmetry $V(x)$ generated by elements $(v(x), E, 0)$ is contained in $\Gamma$. The group $\Gamma$ acts in a natural way on $M$:

$$x^a \rightarrow x(\alpha)^a = x^a + t^a(x).$$

An arbitrary diffeomorphism of the manifold $M$ can be represented in the form (7); therefore, (7) defines a homomorphism of the group $\Gamma$ into the group of diffeomorphisms of the manifold $M$. The kernel of such a homomorphism is the group $V(x) \otimes \Lambda(x)$ where $\Lambda(x)$ is the gauge Lorentz group generated by elements $(0, \Lambda(\omega(x)), x)$.

3. We consider functions $H^\alpha(x, g)$, having the following properties:

1. $H^\alpha(x, g)$ depends smoothly on $x$ and $g$.
2. $H^\alpha(x, 0) = 0 \forall x \in M$. 

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