Within the framework of classical (nonquantum) theory of topological transitions, the problem of singularities is discussed; this is one of the basic obstacles to transition to a quantum description. The features of the solution of this problem for a gravitational field and the fields of the sources are considered. In the first case, the singularity problem may be solved by constructing a Lagrangian that is regular in the vicinity of the topological transition. For gravitational-field sources this method is inapplicable, and therefore it is necessary either to use a mechanism analogous to the mechanism of spontaneous symmetry violation or to introduce additional boundary conditions which ensure regularity of the Lagrangian and the field equations.

Introduction

An unsolved problem of the quantum theory of gravitation is the description of quantum topological transitions [1-5]. Attempts at direct solution of this problem encounter considerable difficulties, and therefore in [6-10] the general properties of classical models of topological transitions were investigated, and the general principles for the construction of a classical scalar-tensor theory of topological transitions were formulated. The results in [6-10] lay the ground for the construction of a quantum theory of topological transitions of a certain class. However, transition to a quantum description still requires the solution of a number of problems; one is the problem of singularities [3], arising because the space-time describing the topological transitions has scalar singularities of curvature in the classical approach [6, 10]. The present work undertakes an analysis of the singularity problem in classical theories of topological transitions and a discussion of possible means for its solution.

Principles for the Construction of a Theory of Topological Transitions

In the scalar-tensor theory of topological transitions, the independent variables describing the gravitational field are the scalar field $\varphi$ and the field of the positive-definite tensor $\tilde{g}_{\alpha\beta}$ [8-10]. The Lorentzian structure of the space-time and the motion of the trial particles and sources are determined in this case by the metric $\tilde{g}_{\alpha\beta}$, related to the fields $\varphi$ and $\tilde{g}_{\alpha\beta}$ as follows

$$\tilde{g}_{\alpha\beta} = \frac{2l_{\alpha}l_{\beta}}{f} - \tilde{g}_{\alpha\beta}, \quad (1)$$

where $l_{\alpha} = \tilde{l}_{\alpha}$, $f = l_{\alpha}l_{\beta} = g_{\alpha\beta}l_{\alpha}l_{\beta} = \tilde{g}_{\alpha\beta}l_{\alpha}l_{\beta}$.

Here and below, the following conventions are used: the covariant derivatives in accordance with the metrics $\tilde{g}_{\alpha\beta}$ and $\tilde{g}_{\alpha\beta}$ are denoted by the symbols (') and ('); the geometric quantities corresponding to the metric $\tilde{g}_{\alpha\beta}$ are denoted by a tilde; the indices of quantities constructed from the fields $\varphi$, $\tilde{g}_{\alpha\beta}$ and their derivatives are shifted using the metric $\tilde{g}_{\alpha\beta}$ and those in other cases using the metric in Eq. (1).

It follows from Eq. (1) that the surface of the level $\varphi = $ const is space-like, while the vector field $l_{\alpha} = \varphi_{,\alpha}$ is time-like. If some hypersurface of the level $\varphi = a = $ const contains a critical point of the function $\varphi$, i.e., a point at which $\varphi_{,\alpha} = 0$, the surfaces of the levels $\varphi = a - \epsilon$ and $\varphi = a + \epsilon$, where $\epsilon = $ const $> 0$, may have a different topology [11]. This means that theories with independent variables $\varphi$ and $\tilde{g}_{\alpha\beta}$ may be regarded as scalar-tensor theories of topological transitions, and permit the construction of models of topo-
logical transitions [7] giving the topological realization of the idea of a Planck vacuum [12, 13] and quantum creation of the Universe [14]. The correctness of this approach follows from the independence of the types of topological transitions from the specific representation of the metric $\tilde{g}_{\alpha\beta}$ in the form in Eq. (1) [8, 9].

The critical points of the function $\phi$ are, first, the significantly singular points of the metric in Eq. (1) and, second, the scalar singularities of curvature of the space-time [6, 8-10]. In contrast to general relativity theory, this does not lead to difficulties associated with determining the structure of the manifold in the vicinity of the singular points, if the metric $\tilde{g}_{\alpha\beta}$ is regular. However, there remains the problem of extending the action integral and field equations at the singular points, in that the Lagrangian of the sources $L_s$ is of the same form as in general relativity theory and, for this reason, contains a pole of finite order and significantly singular points, while the gravitational Lagrangian $L_g$ may contain invariants of curvature of the metric in Eq. (1), as in the modification of general relativity theory and Branco–Dicke theory considered in [8-10], for example.

**Regularization of Gravitational Field**

The general principles for constructing metric theories of gravitation [15] do not impose any constraints on the dependence of the gravitational Lagrangian $L_g$ on the fields $\tilde{g}_{\alpha\beta}$ and $\phi$. Therefore, the regularization problem for a gravitational field may be solved by two methods: by direct construction of a regular Lagrangian from the variables $\phi$ and $\tilde{g}_{\alpha\beta}$, or by regularization of the Lagrangian from any existing theory.

In the first case, the Lagrangian $L_g$ must not only satisfy the regularization conditions but also some condition that it be "natural," deriving from field-theory and group considerations and, further, ensure the correct post-Newtonian approximation. The construction and investigation of such a Lagrangian must form the subject of a separate work. Therefore, attention is confined here to the second method.

Consider the Lagrangian

$$L_g = \frac{R}{1 + f} + 4f^2 l^2 \tilde{R} + 2 \left( l^4 l^2 - l^2 \right).$$

When $f = 1$, the Lagrangian in Eq. (2) coincides with the Einstein–Hilbert Lagrangian of general relativity theory, which takes the following form for the metric in Eq. (1) [10]:

$$L_{E H} = \frac{R}{1 + f} + \frac{1}{f} \left( 4l^2 l^2 \tilde{R} + 2 \left( l^4 l^2 - l^2 \right) - 2f l^2 \right) +$$

$$+ \frac{\phi}{f^2} \left( 2l^4 l^2 \phi + 2f l^2 \phi + l^4 l^2 \phi \beta \right) - 4 \left( l^4 l^2 \right)^2 \phi^2 - 4 \phi \beta .$$

Hence, in the class of variations satisfying the condition $f = 1$, the Lagrangian in Eq. (2) leads to the same field equations as that in Eq. (3), i.e., to equations of the scalar-tensor modification of general relativity theory [8-10], whereas for variations of general form nonequivalent systems are obtained.

The equivalence of theories with the Lagrangians in Eqs. (2) and (3) when $f = 1$, which may readily be extended also to other versions of gravitational theory, may be regarded as some variational "equivalence principle," since the condition $f = 1$ may be easily satisfied outside the vicinity of the critical points of the function $\phi$ by renormalizing the fields $\phi$ and $\tilde{g}_{\alpha\beta}$. This renormalization is legitimate since the fields $\phi$ and $\tilde{g}_{\alpha\beta}$ appear in the Lagrangian of sources only in the form in Eq. (1), and hence the motion of the sources is determined by the class of equivalent pairs $(\phi, \tilde{g}_{\alpha\beta})$ specifying the same Lorentzian metric $g_{\alpha\beta}$ at $M^4$.

**Regularization Problem for Sources**

According to general principles [15], sources of gravitational field interact directly only with the metric $\tilde{g}_{\alpha\beta}$ related to the fields $\phi$ and $\tilde{g}_{\alpha\beta}$ by Eq. (1). Therefore, the methods described above for regularization of the gravitational field are not applicable for the fields of sources without significant changes in the basic principles of the theory.

Two methods may be proposed for regularization of the Lagrangian of sources, remaining in the form of relations of the sources with gravitation in the regular region. The first method is to introduce an additional scalar field $\chi$ in the metric $g_{\alpha\beta}$ so that Eq. (1) takes