THE PSEUDOTURBULENT DIFFUSION OF PARTICLES IN HOMOGENEOUS SUSPENSIONS

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The effective diffusion coefficients for suspended particles caused by their pseudoturbulent pulsations, are treated. Derivatives of the dynamic variables which determine the average motion of the locally homogeneous suspension are neglected.

By definition [1] the tensor of particle diffusion coefficients, for the case in which the diffusion is brought about by the random motion of particles, can be represented in the form

\[ D_{ij} = \frac{1}{2} \int_0^\infty (R_{wi, wj}(\tau) + R_{wi, wj}(\tau)) d\tau \]  

(1)

where the integrand consists of components of the tensor of Lagrangian correlation functions for the particle velocity \( \mathbf{w} \). These quantities can be written in the form

\[ R_{wi, wj}(\tau) = \int e^{i\mathbf{w} \cdot \mathbf{k}} \mathbf{w}_{wi, wj}(\omega, \mathbf{k}) d\omega d\mathbf{k}. \]  

(2)

Here the integration is carried out over all frequencies \( \omega \) and all wave-space \( \mathbf{k} \), while \( \mathbf{w}_{wi, wj}(\omega, \mathbf{k}) \) is the spectral tensor of the random vector \( \mathbf{w} \), introduced in [2]. This tensor is defined in [2] in such a way that its integral with respect to wave-space is the ordinary Lagrangian spectral tensor of particle velocity.

It can easily be seen from [2] that the tensor \( \mathbf{w}_{wi, wj}(\omega, \mathbf{k}) \), considered as a function of \( \omega \), satisfies all the conditions necessary for changing the order of integration with respect to \( \tau \) and \( \omega \) in (1) and (2). Changing the order of integration and using the Fourier integral expansion for the \( \delta \)-function, we obtain the following relation for the pseudoturbulent particle diffusion tensor from (1) and (2):

\[ D_{ij} = \pi \int \mathbf{w}_{wi, wj}(0, \mathbf{k}) \cdot \mathbf{w}_{wi, wj}(0, \mathbf{k}) d\mathbf{k}. \]  

(3)

The usual means [1] of expressing the quantities \( \mathbf{w}_{wi, wj}(\omega, \mathbf{k}) \) is in terms of average products of corresponding components of the spectral measure \( dZ \mathbf{w} \) of the random process \( \mathbf{w} \). Equations for \( dZ \mathbf{w} \) and spectral measures of other random quantities characterizing pseudoturbulence in a suspension are obtained in [2]. It can easily be seen from (3) that in the present paper we have only to consider these equations for zero frequency \( \omega \) and only for a steady-state flow without gradient, when the scales of the average motion are much longer than the scale of the pseudoturbulence, i.e., when we can neglect derivatives of dynamic variables characterizing suspension flow in the continuous approximation. It was shown in [2] that this latter corresponds to the familiar Euler approximation in the hydrodynamic approximation of a single-phase fluid. For \( \omega = 0 \) we have the following equations from [2]:
Here the same symbols are adopted as in [2], but the average sign $\langle \rangle$ is omitted from the symbols for dynamic variables to simplify the notation. In the derivation of (4) an expression was used for the interaction force between phases, valid for $R \ll 1$, where $R = 2 au/v_0$ is the Reynolds' number. This expression characterizes the relative flow of the liquid phase around individual particles.

It is convenient to pass to dimensionless variables, introducing a characteristic velocity $u$, characteristic length $a$, and consequently a characteristic time $a/u$. In what follows, the treatment is presented for dimensionless variables, which are the ratios of the corresponding variables to dimensional quantities constructed from the characteristic scales selected. Thus, for example, the dimensional velocities and diffusion coefficients are divided by $u$ and $ua$ respectively, the dimensional frequency and wave-vector by $u/a$ and $1/a$, etc. The only exception is the dimensional spectral measure of pressure perturbations in the flow $dZp$, which is divided by $d_0 auK$. When quantities are rendered dimensionless in this way, the form of Eqs. (1)-(3) is retained, and instead of (4) we have

$$d\ln K - \frac{kZo}{1 - \rho} dZp + (1 + \frac{1}{2}) dZo = \frac{kZp}{1 - \rho} dZo - \frac{kZo}{1 - \rho} dZo = \frac{kZo}{1 - \rho} dZo$$

The parameter $\alpha$ in (5) characterizes the ratio of dissipative forces arising from the instantaneous acceleration of the associated fluid mass due to instantaneous velocity changes of colliding particles, to the viscous interaction forces between phases [2]. An order-of-magnitude estimate was obtained in [2] for $\alpha$ on the basis of a model in which there are elastic collisions in a gas of particles having an isotropic Maxwell velocity distribution. In dimensionless form this gives us

$$\alpha = \frac{1}{3K} \left[ \frac{3}{\pi} \left( \frac{1}{\rho R} \right)^{\frac{1}{2}} \left( \frac{1}{\rho R} \right)^{\frac{1}{2}} \left( \frac{1}{\rho R} \right)^{\frac{1}{2}} \right]$$

The symbols of paper [2] are also retained here.

From physical considerations it is natural to expect that "collisional" dissipation in a dispersive system is relatively small, i.e., $\alpha \ll 1$, at least for systems in which the concentration is not very close to the concentration of a granular layer in the close-packed state. This conclusion results from the following considerations in particular.

1. By its nature collisional dissipation is proportional to the collision frequency in the suspension of and the size of velocity discontinuity for colliding particles, i.e., it is always small for rarefied suspensions.

2. The collisions of particles suspended in the fluid usually lead to quite smooth, rather than abrupt, changes of particle velocity. This is associated with the considerable pressure increase in the fluid layer between particles as they approach each other, and the necessity for "squeezing out" this layer before there can be direct contact of the particles. A similar effect also occurs when a particle approaches a solid wall [3], and in lubrication processes, when the part of the fluid layer is played by the lubricating fluid in the space between the journal and bearing [4]. We can thus assume that the estimate (5) based on a model of purely elastic collisions between particles, is higher by an order of magnitude even for suspensions which are not very concentrated.

3. Finally we can expect a substantial effect from direct particle collisions (contacts) predominantly in suspensions of coarse and heavy particles in fluids of low density and viscosity, particularly in gases.