TEMPERATURE FIELDS AND STRESSES IN A SYSTEM
OF PLANE-PARALLEL LAYERS UNDER HEATING
BY ELECTROMAGNETIC RADIATION

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We determine and study the temperature fields and stresses arising in heating of a system of two plane-
parallel layers of different transparency by electromagnetic radiation. We discuss the possibility of using
the results obtained to develop rational programs for thermal processing of electrovacuum devices with a
glass shell. Three figures. Bibliography: 8 titles.

We consider a system consisting of layers (1) and (2) separated by a vacuum, of thickness $h_1$ and $h_2$, under
the action of an external electromagnetic radiation, and semitransparent and nontransparent respectively
to the radiation. The radiation comes from a source in the direction of layer (1). On the outer face of layer
(1) the condition of convective heat exchange with the environment holds and on the outer surface of layer
(2) the condition of both heat exchange by radiation and convective heat exchange with the environment
that is transparent to the radiation (the temperatures of the external environments are assumed to be $T_81$
and $T_82$ respectively). The layers (1) and (2) are not mechanically connected, and their end sections may
be rigidly clamped or free of any force load.

We pose the problem of determining the temperature field and stresses in each of the layers. We
shall connect the thermally stressed state of the body under the action of electromagnetic radiation with
the heat liberation resulting from the absorption of radiation energy by the material. Then when the
characteristics of the material of the layers is constant, finding the unknown stressed state reduces to
determining the radiation field, the temperature field, and the stresses in the system. In this situation in
the semitransparent layer we determine the intensity of radiation and the heat liberation corresponding
to it from the transfer equations in the approximation of nonradiating material [4, 7]. We use these heat
liberation as the specific power of the continuously distributed heat sources in the heat equation. For the
nontransparent layer, because of surface absorption or radiation of electromagnetic energy, we shall take
account of the radiation intensity only in the boundary condition of balance of heat flow on the surface.
After determining the temperature field from the thermoelastic equations, we find the stress-deformed state
of the layers caused by it. We take account of the action of the source of electromagnetic radiation on
the system by prescribing the radiation flow incident on the outer surface of layer (1), the intensity $i_{\lambda 0}$ of
which is determined by the radiation properties of the source.

We introduce a Cartesian coordinate system { $x_1, x_2, z$ } so that the plane $z = 0$ coincides with the
middle plane of layer (2) and the outside face of layer (1) is at distance $l$ from it. In accordance with the
computation scheme introduced above, after determining the radiation field in the semitransparent layer
(1), we find the specific power of the heat liberation in it. In doing this we can write the expression for the
heat liberation [4,7] in the following form:

\[ Q = 2\pi \int_0^\infty a_\lambda \left[ \int_0^{\pi/2} i^+_\lambda(z, \beta) \sin \beta d\beta + \int_{\pi/2}^{\pi} i^-_\lambda(z, \beta) \sin \beta d\beta \right] d\lambda. \]  

Here

\[ i^+_\lambda(z, \beta) = i^+(\beta) \exp[-a_\lambda(z - l + h_1) \cos^{-1} \beta], \]

\[ i^-_\lambda(z, \beta) = i^-(\beta) \exp[a_\lambda(l - z) \cos^{-1} \beta], \]  

where \( i^+ \) and \( i^- \) are the intensities of effective radiation on the surfaces \( z = l - h_1 \) and \( z = l \) of layer (1) in the direction of propagation of the ray, forming an angle \( \beta \) with the \( z \)-axis; \( a_\lambda \) is the coefficient of absorption of the material of the layer, which is a function of the wave length \( \lambda \) of the radiation.

The expression for the flux of electromagnetic energy absorbed by the nontransparent layer (2) through the surface \( z = h_2/2 \) is [7]

\[
q = \pi \int_0^\infty \varepsilon_\lambda \left[ \int_{\pi/2}^\pi i_{\lambda*}^- (\beta) \sin \beta \, d\beta \right] d\lambda, \tag{3}
\]

where \( \varepsilon_\lambda \) is the spectral degree of blackness of the material of layer (2), and \( i_{\lambda*}^- \) is the intensity of the directed effective radiation on the surface \( z = h_2/2 \).

In expressions (1) and (3) the radiation intensities on the surfaces of the layers, which are assumed to be diffusively reflecting in what follows, are determined from the balance conditions for the fluxes of incident, reflected, and intrinsic radiation on these surfaces:

\[
i^- (\mu) = 2 \rho_\lambda \int_0^1 i^+ (\mu_0) \exp(-a_\lambda \mu_0^{-1}) \, d\mu_0 = n_\lambda^2 (1 - \rho_\lambda) (i_{\lambda0}^- (\pi - \xi)),
\]
\[
i^+ (\mu) = -2 \rho_\lambda \rho_\lambda^2 n_\lambda^2 \int_0^1 i^+ (\mu_0) \mu_0 \, d\mu_0 - 2 \rho_\lambda (1 - \rho_\lambda \rho_\lambda^2) \int_0^1 i^- (\mu_0) \exp(-a_\lambda \mu_0^{-1}) \, d\mu_0
\]
\[
- 2(1 - \rho_\lambda^2) \rho_\lambda \int_\mu^1 i^- (\mu_0) \mu_0 \exp(-a_\lambda \mu_0^{-1}) \, d\mu_0 = n_\lambda^2 (1 - \rho_\lambda) i_{\lambda2}^+ (\xi), \tag{4}
\]
\[
i_{\lambda*}^- (\mu) = 2 \rho_\lambda \rho_\lambda^2 \int_0^1 i_{\lambda*}^- (\mu_0) \mu_0 \, d\mu_0 = -\rho_\lambda i_{\lambda2}^+ (\mu) + n_{\lambda}^{-2} (1 - \rho_\lambda) i^- (\eta) \exp(x_\lambda \eta^{-1}).
\]

These conditions are obtained taking account of the mutual dependence of the fluxes under consideration for diffusively reflecting bodies. Here

\[
\mu = \cos \beta, \quad \xi = \arcsin n_\lambda \sqrt{1 - \mu^2}, \quad \eta = -\sqrt{1 - (1 - \mu^2) n_{\lambda}^{-2}}, \quad \mu_* = \cos \beta_*;
\]

\( \rho_\lambda \) and \( \rho_\lambda^2 \) are the coefficients of diffusive reflection of the layers (1) and (2); \( n_\lambda \) and \( x_\lambda = a_\lambda h_1 \) are respectively the index of refraction and the optic thickness of layer (1); \( \beta_* = \arcsin n^{-1}_\lambda \) is the maximal angle of refraction of a ray; and \( i^+_{\lambda2} \) is the intensity of intrinsic radiation from the surface \( z = h_2/2 \) of layer (2).

From the system (4) we find the following expressions for the intensity on the surfaces, written in terms of the intensities \( i_{\lambda0}^- \) and \( i_{\lambda2}^+ \) of the external radiation and intrinsic radiation of the surface

\[
i^+ (\mu) = N_1 i_{\lambda2}^+ (\xi) + N_2 i_{\lambda*}^+ (\mu) + N_3 f_*,
\]
\[
i^- (\mu) = N_4 i_{\lambda2}^+ (\mu) + N_1 i_{\lambda0}^- (\pi - \xi) + N_5 f_*,
\]
\[
i_{\lambda*}^- (\mu) = N_6 i^- (\eta) \exp(x_\lambda \eta^{-1}) + N_7 i_{\lambda2}^+ (\mu) + N_8 f_*, \tag{5}
\]

where

\[
f_* = \int_\mu^1 \mu_0 i_{\lambda0}^- (\pi - \xi) \exp(-x_\lambda \mu_0^{-1}) \, d\mu_0; \quad N_1 = n_{\lambda}^2 (1 - \rho_\lambda), \quad N_2 = -2 N_1 A_1 (\lambda) B_1 B_2, \quad N_3 = -N_2 B^2_2, \quad N_4 = 2 \rho_\lambda \left[n_\lambda^2 A_3 (\lambda) + E_3 (x_\lambda) N_2 \right], \quad N_5 = 4 \rho_\lambda E_3 (x_\lambda) A_1 (\lambda) B_1 N_1, \quad N_6 = n_{\lambda}^{-2} (1 - \rho_\lambda),
\]
\[
N_7 = B_2 \left[1 - 4 \rho_\lambda \rho_\lambda^2 (1 - \rho_\lambda) A_3 (\lambda) + (1 - \rho_\lambda) B_4 \right], \quad N_8 = 2 \rho_\lambda B_3 (1 - \rho_\lambda^2) (1 - 2 \rho_\lambda B_4);
\]
\[
B_1 = [1 + A_4 (\lambda) - A_2 (\lambda) E_3 (x_\lambda)]^{-1}, \quad B_2 = A_2 (\lambda) A_3 (\lambda) - A_4 (\lambda), \quad B_3 = \rho_\lambda (1 - \rho_\lambda \rho_\lambda^2)^{-1}, \quad B_4 = 2 A_1 (\lambda) A_3 (\lambda) E_3 (x_\lambda) B_1; \quad A_1 (\lambda) = \rho_\lambda + \rho_\lambda - 2 \rho_\lambda \rho_\lambda^2,
\]
\[
A_2 (\lambda) = 4 \rho_\lambda A_1 (\lambda) E_3 (x_\lambda) - \mu_\lambda^2 (1 - \rho_\lambda^2) \rho_\lambda E_3 (x_\lambda \mu_*^{-1}), \quad A_3 (\lambda) = E_3 (x_\lambda) - \mu_*^2 E_3 (x_\lambda \mu_*^{-1}), \quad A_4 (\lambda) = \rho_\lambda \rho_\lambda n_{\lambda}^2 (1 - \mu_*^2);
\]