being the analog of Levy's representation \( \mu(a, \sigma; M, N) \) of the characteristic function of the infinitely divisible law.

The representation (9) has the following properties. Let \( \alpha \) be the index of the GGSD with characteristic function (9), introduced in Theorem 3. Then

1) if \( \alpha \neq 2 \), then \( \sigma = 0 \);
2) if \( \alpha = 2 \), then \( M = N = 0 \);
3) if \( \alpha > 2 \) for \( \alpha < 0 \), then \( \sigma = 0, M = N = 0 \) and, therefore, \( \phi(1) = 1 \);
4) if \( h(u) = u^\alpha N(u) \) and \( k(u) = |u|^\beta M(u) \), then \( h \) and \( k \) satisfy the functional equations

\[
h(u) = \sum \alpha_i \xi_i^\alpha h\left(\frac{\xi_i}{\xi_i^\alpha}\right), \quad k(u) = \sum \beta_i \xi_i^\beta k\left(\frac{\xi_i}{\xi_i^\beta}\right).
\]

The proofs of these properties are obtained in the same way as for \( \exp\left(1-1/s(\xi)\right) \) in [3].

**LITERATURE CITED**


**CONSTRUCTION OF RANDOM VARIABLES ACCORDING TO THE PROBABILITIES OF THEIR BINARY DIGITS**

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One states special properties of binary expansions of random variables, having a "truncated" geometric (discrete uniform) and exponential (uniform) distributions, which simplify in an essential manner the construction of random variables according to the probabilities of their binary digits.

As mentioned in [2], the imitation of "randomness" has become a useful and frequently applied tool in mathematical statistics and this circumstance establishes the interest in the construction of random variables (r.v.) with the aid of computers. There exists a close connection between that part of analytic statistics which we call "the arithmetic of binary expansions of r.v." and the methods of construction of r.v. This connection will be revealed in this communication.

At some time, the method of the construction of r.v. according to the conditional probabilities of their binary digits has been popular (see, for example, [8]). In this paper, on the examples of the "truncated" geometric (discrete uniform) and exponential (uniform) distributions, it is shown that the use of the remarkable properties of the binary expansions of the r.v. having these distributions can simplify the method in an essential manner, with-

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out requiring additional computer memory for the storage of the tables of the conditional probabilities.

Assume that the r.v. \( \eta = \sum_{0 \leq i < l} q^{(i)} 2^i \), where \( q^{(i)} \) are equal to 0 and 1, have the "truncated" geometric distribution

\[
P(\eta = k) = (1-q)q^k/(1-q^{k+1}), \quad k = 0, 1, \ldots, n
\]

for \( 0 < q < 1 \) and \( 2^{l-1} < n < 2^l \), \( l > 1 \). It turns out that if for some \( i \) the conditional probability

\[
P(\eta^{(i)} = 0 \mid \eta^{(i+1)} = k^{(i+1)}, \ldots, \eta^{(l-1)} = k^{(l-1)}) \neq 1, \quad l \leq i \leq l,
\]

then for all successive binary digits we have

\[
P(\eta^{(i)} = 1 \mid \eta^{(i+1)} = k^{(i+1)}, \ldots, \eta^{(l-1)} = k^{(l-1)}) = q^{2i}/(1 + q^{2i}),
\]

i.e., \( \eta^{(i)} \) for \( 0 \leq i < l \) are mutually independent.

Indeed, since the previous condition means that the \( l \)-digit integer \( k_{\{i\}} = (k^{(l-1)} \ldots k^{(i)} 1 \ldots 1)_2 < n \), it follows that

\[
P(\eta^{(i)} = 1 \mid \eta^{(i+1)} = k^{(i+1)}, \ldots, \eta^{(l-1)} = k^{(l-1)}) =
\[
= \sum_{k = k_{\{i\}} - 2^{l-1} + 1}^{k_{\{i\}}} q^k / \sum_{k = k_{\{i\}} - 2^{l-1} + 1}^{k_{\{i\}}} q^k = q^{2i}/(1 + q^{2i})
\]

and in the same manner one obtains the remaining ones.

In particular, for \( n = 2^l - 1 \) all the binary digits \( \eta^{(i)} \) of the r.v. \( \eta \) are mutually independent and

\[
P(\eta^{(i)} = 1) = 1 - P(\eta^{(i)} = 0) = q^{2i}/(1 + q^{2i}), \quad 0 \leq i < l.
\]

From here it follows that if the r.v. \( \eta = \sum_{i \geq 0} q^{(i)} 2^i \) assumes an infinite number of values with probabilities \( P(\eta = k) = (1-q)q^k, \quad k > 0 \), then all \( q^{(i)} \) are mutually independent. The converse is also true, namely, if the r.v. \( \eta \) with independent binary digits is unbounded, then the "truncation" of \( \eta \) to a finite set of values implies the independence of its binary digits, starting with some index.

For \( q = 1/2 \) the bounded r.v. \( \eta \) is distributed uniformly on the set \( \{0, 1, \ldots, n\} \) and, by analogy with the previous case, if some digit in the binary representation of \( \eta \) turns out to be equal to 0 and the conditional probability of the equality to zero of the given digit is different from 1, then the subsequent digits of the r.v. \( \eta \) are mutually independent and for each \( n \), such that \( 2^{l-1} \leq n < 2^l \), \( l > 1 \), assume the values 0 and 1 with probability 1/2 (in the case \( n = 2^l - 1 \) all the binary digits \( \eta^{(i)} \) of the r.v. \( \eta \) are independent and \( P(\eta^{(i)} = 1) = P(\eta^{(i)} = 0) = 1/2, \quad 0 \leq i < l \)).

Assume further that the r.v. \( \xi \) has an exponential distribution with parameter \( \lambda > 0 \). Since the integer part of the r.v. \( \xi \) has a geometric distribution with \( \xi = e^{-\lambda} \), we restrict