A physical interpretation of the parameters present in the generalized Kerr—NUT solution is given. A theory of the multiple structure of gravitational sources, developed within the limits of the Newman—Penrose formalism, was utilized for this purpose. The results obtained introduce several new moments into the interpretation of the solution under consideration.

In a previous paper [1], we considered the solution of Einstein's equations, obtained by Demianski and Newman (DN) [2], which generalized the well-known "charged" Kerr [3] and Newman—Unti—Tambarino (NUT) [4] metrics. We showed that the current interpretation of the parameters present in the solution (DN) is not conclusive and contains certain obscurities. The means which serve as the basis for a physically convincing interpretation of the parameters indicated involves the construction of a gravito-inertial reference frame (gravi-IRF) in the space considered and a Bondi coordinate system rigidly connected to it [5, 6, 7]. In order to understand the following, knowledge of certain facts concerning the Newman—Penrose formalism [9] appears to be essential. We shall give a brief account of them here.

The method of Newman—Penrose (NP) is based on assuming the existence of four isotropic vector fields forming a quasiorthogonal tetrad

\[ Z_{\mu\nu} = (l_\mu, n_\mu, m_\mu, \bar{m}_\mu), \quad g_{\rho\sigma} = 2 \left[ l_\rho n_\sigma - m_\rho \bar{m}_\sigma \right] \]  

in a selected region of a manifold. Two of the vectors here are real and two are complex, mutually conjugate, vectors. The vectors of the tetrad (1) satisfy the following orthogonality relations:

\[ \begin{align*}
l_\mu l^\nu &= n_\mu n^\nu = m_\mu m^\nu = \bar{m}_\mu \bar{m}^\nu = 0, \\
l_\mu m^\nu &= l_\mu \bar{m}^\nu = n_\mu m^\nu = n_\mu \bar{m}^\nu = 0, \\
l_\mu n^\nu &= -m_\mu \bar{m}^\nu = 1.
\end{align*} \]  

However, conditions (2) only determine the vectors \( l_\mu, n_\mu, m_\mu, \) and \( \bar{m}_\mu \) correct to within the restricted, six-parameter Lorentz group, which decomposes into three subgroups [10]:

a) two-parameter, zeroth-order rotations about \( n_\mu \),

\[ \tilde{l}^\nu = l^\nu + b \bar{m}^\nu + \bar{b} m^\nu + b \bar{b} m^\nu, \quad \tilde{n}^\nu = n^\nu, \quad \tilde{m}^\nu = m^\nu + b n^\nu; \]  

b) two-parameter, zeroth-order rotations about \( l_\mu \),

\[ \tilde{n}^\nu = n^\nu + a \bar{m}^\nu + \bar{a} m^\nu + a \bar{a} l^\nu, \quad \tilde{l}^\nu = l^\nu, \quad \tilde{m}^\nu = m^\nu + a l^\nu; \]  

c) homogeneous Lorentz transformations in the \( l^\mu, n^\mu \) plane and spatial rotations in the \( m^\mu, \bar{m}^\mu \) plane,

\[ \tilde{l}^\nu = G l^\nu, \quad n^\nu = G^{-1} n^\nu, \quad \tilde{m}^\nu = e^{iH} m^\nu, \]  

where \( G \) and \( H \) are real.

Instead of the Ricci rotation coefficients \( \gamma_{\mu\nu\rho} = Z_{\mu\nu} \gamma^\rho \), their linear combinations, defined in the following manner:


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are utilized in the NP formalism. The tetradic components of the Weyl tensor are expressed in terms of five complex scalars,

\[
\Psi_0 = -C_{\alpha \beta \gamma} l^\alpha m^\beta l^\gamma m^\gamma, \quad \Psi_1 = -C_{\alpha \beta \gamma} l^\alpha n^\beta l^\gamma m^\gamma, \\
\Psi_2 = -\frac{1}{2} C_{\alpha \beta \gamma} (l^\alpha n^\beta l^\gamma n^\gamma - l^\alpha n^\beta m^\gamma m^\gamma), \\
\Psi_3 = -C_{\alpha \beta \gamma} n^\alpha l^\beta n^\gamma m^\gamma, \quad \Psi_4 = -C_{\alpha \beta \gamma} n^\alpha m^\beta n^\gamma m^\gamma.
\]

The operators

\[
D = l^\alpha \frac{\partial}{\partial x^\alpha}, \quad \Delta = n^\alpha \frac{\partial}{\partial x^\alpha}, \quad \delta = m^\alpha \frac{\partial}{\partial x^\alpha}, \quad \bar{\delta} = \bar{m}^\alpha \frac{\partial}{\partial x^\alpha}
\]

are introduced to designate inner derivatives. One can introduce two types of coordinate systems associated with the tetrad (1).

Type I is characterized by the fact that we select isotropic geodesics, tangent to the vector \( l^\mu \), for the reason that it is orthogonal to the hypersurfaces \( u = \text{const} \). These isotropic hypersurfaces are selected as the coordinates \( x^\alpha \). The affine parameter along each geodesic serves as the radial coordinate \( x_2 = r \). Since the hypersurface is orthogonal, we can choose to expand the congruency \( \rho = -l^\mu \rho_\mu / 2 \) in the form

\[
\rho = -r^{-1} + O(r^{-3}).
\]

The complex shear of the congruency has the form

\[
\sigma = \phi^\beta r^{-2} + O(r^{-4}).
\]

The subsequent simplification is connected with the fact that the freedom in the selection of the vectors \( n^\mu \) and \( m^\mu \), specified by Eqs. (4) and (5), was preserved. We require that they should be transposed parallel along \( l^\mu \). Everything stated above reduces to the following limitations on the coefficients (NP) and the tetrad of vectors (1)

\[
\kappa = \tau = \epsilon = 0, \quad p = \bar{p}, \quad \tau = \bar{\tau}, \quad \bar{\rho} = \tilde{\rho}, \quad \rho = \bar{\rho}, \\
\lambda = \lambda, \quad \sigma = \sigma, \quad \bar{\sigma} = \tilde{\sigma}, \\
\lambda = \lambda, \quad \sigma = \sigma, \quad \bar{\sigma} = \tilde{\sigma},
\]

The two remaining coordinates \( x^2 \) and \( x^3 \) are selected in such a manner that they label the geodesics on spheres generated by the intersection of the hypersurfaces \( u = \text{const} \) and \( r = \text{const} \). (The requirement that just spheres should be obtained in the intersection reduces to the condition \( \gamma^0 = 0 \) [11].) A similar type of coordinate system was first considered by Bondi (see, for example, [7]). It has been utilized in the basic papers [9, 12].

In order to construct a coordinate system of the second type, we shall select isotropic geodesics with the tangent vector \( \tilde{l}^\mu \) for the reason that it will not be orthogonal to the hypersurfaces \( u = \text{const} \), which again are selected as the \( x^\alpha \) coordinate. The affine parameter along the geodesics \( \tilde{\tau} \), so that \( \tilde{l}^\mu = \delta^\mu_\nu \tilde{\tau} \), are again taken to be the second coordinate. The freedom in the selection of \( \tilde{\tau} \) allows one to write the expansion of the congruency in the form

\[
\tilde{\rho} = -(r + l \Sigma)^{-1} + O(r^{-3}), \quad \Sigma = a \cos \theta + b.
\]

The shear is now defined as

\[
\tilde{\sigma} = O(r^{-3}).
\]

The \( x^2 \) and \( x^3 \) coordinates again label the geodesics at the intersection of the hypersurfaces \( u = \text{const} \) and \( r = \text{const} \). Thus, in the given coordinate system, the geodesic rays are asymptotically infinite, although, generally speaking, they are rotated. A coordinate system of this type was utilized, for example, by Talbot [13]. Any asymptotically plane solution of Einstein's equations (or the Einstein—Maxwell equations) can be written in both the first and second type of coordinate system [14]. The important point, however,