A methodology is proposed for the numerical construction of an optimal control for a nonlinear system in the presence of nonclassical constraints. The sequential linearization and reduction of all the constraints of the problem to a single integral form allow one on the different stages of solution of the problem to use their discretization step and thereby to preserve the dimension of the resulting nonlinear programming problem.

Let the behavior of the mechanical system be described by the vector differential equation

\[
\frac{dx}{dt} = f(x, u, t), \quad x(0) = x^0;
\]

where \( x(t) \) is the phase vector of the system; \( u(t) \) is the vector of controls; \( T' \) is the nonfixed moment of completion of the control process; the function \( f: \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R}^1 \to \mathbb{R}^n \) is continuously differentiable with respect to \( x \) and \( u \). The criterion of quality of control is the terminal functional

\[
\Phi = \Phi(x(T'), T').
\]

It is required to construct a continuous control \( u(t) \) taking system (1) from the state \( x^0 \) to the terminal manifold

\[
h(x(T'), T') = h_f.
\]
and minimizing the functional (2) under the constraints

\[ \varphi (u(t), t) \leq 0; \]
\[ \psi (x(t), t) \leq 0, \quad \forall t \in [0, T']. \]  

(4)

Here the functions \( \Phi : \mathbb{R}^n \to \mathbb{R}^l \) and \( h : \mathbb{R}^n \to \mathbb{R}^k \) are continuously differentiable with respect to \( x(T') \) and \( T' \), the function \( \varphi : \mathbb{R}^l \to \mathbb{R}^m \) is continuously differentiable with respect to \( u(t) \), and the function \( \psi : \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}^l \) is continuously differentiable with respect to \( x(t) \). Below for simplicity let \( m = l = 1 \).

Making the change of the independent variable \( t = \omega \tau \), let us move to a problem with fixed time \( T = T'/\omega = \text{const} \) and the supplementary control parameter \( \omega \). Taking into account the substitution, the relations (1)-(4) assume the form

\[ \frac{dx}{d\tau} = \omega f(x, u, \omega \tau), \quad x(0) = x^0; \]  
(5)

\[ \Phi = \Phi(x(\omega \tau), \omega \tau) \rightarrow \min; \]  
(6)

\[ h(x(\omega \tau), \omega \tau) = h^0; \]  
(7)

\[ \varphi (u(\omega \tau)) \leq 0; \quad \forall \tau \in [0, T]; \]  
(8)

\[ \psi (x(\omega \tau), \omega \tau) \leq 0. \]  
(9)

Let \( x_0(\tau) \) be the solution of the Cauchy problem (5) corresponding to some admissible control \( u_0(\tau) \) and satisfying for \( \omega = \omega_0 \) the conditions (7)-(9). To construct variations of the control \( \delta u_0(\tau) \) and parameter \( \delta \omega_0 \) improving the functional (6), let us linearize (5) in the neighborhood of \( x_0, u_0, \omega_0 \)

\[ \frac{d\delta x_0}{d\tau} = \omega f_x \delta x_0 + \omega f_u \delta u_0 + \omega \left( f_\omega + \frac{1}{\omega_0} f_\omega^0 \right) \delta \omega_0; \]  
(10)

\[ \delta x_0(0) = 0. \]

Let us represent the corresponding variation of (6) in the form

\[ \delta \Phi_0 = \Phi_{x(\tau)} \delta x_0(\tau) + \Phi_{u(\tau)} \delta u_0. \]  
(11)

Here \( f_x, f_u, f_\omega, \Phi_{x(\tau)}, \Phi_{\omega} \) are the Jacobi matrices with dimensions \( n \times n, n \times r, n \times 1, 1 \times n, 1 \times 1 \).

Let us break the interval \([0, T]\) into \( N \) intervals \( \Delta \tau \). Let \( \tau_j \in [0, T] \) \((j = j_1, j_2, ..., j_N)\) be the discrete moments of time at which the constraint (8) is active, and \( \tau_i \in [0, T] \) \((i = i_1, i_2, ..., i_p)\) be the analogous moments of time for the constraint (9). To form the system of constraints imposed on the variations of the control \( \delta u_0(\tau) \), the parameter \( \delta \omega_0 \), and the phase variables \( \delta x_0(\tau) \), let us linearize (7)-(9), where in the relations (8)-(9) we consider only the active components

\[ h_{x(\tau)} \delta x_0(\tau_1) + h_{u(\tau)} \delta u_0(\tau_1) = 0; \]  
(12)

\[ \psi_{x}(\tau_1) \delta x_0(\tau_1) + \psi_{u}(\tau_1) \delta u_0 + \psi_{\omega}(\tau_1) \delta \omega_0 = 0, \]  
(13)

(14)

where \( h_{x(\tau)}, h_{u}, \varphi_{u}, \psi_{x}, \varphi_{\omega}, \psi_{\omega} \) are the Jacobi matrices of the corresponding dimensions. In this manner, the general number of constraints (12)-(14) is \( e = k + s + p \).

Let us exclude the variation of the phase variables \( \delta x_0(\tau) \) from relations (11)-(14), for which we will represent the solution of the equation in the variations (10) in the Cauchy form

\[ \delta x_0(\tau) = \omega_0 \int_{\tau}^{\tau_1} h_{u}(s, \omega_0) \delta u_0(s) ds + \omega_0 \delta \omega_0 \int_{\tau}^{\tau_1} h_{\omega}(s, \omega_0) \delta \omega_0 + \int_{\tau}^{\tau_1} f_\omega + \frac{1}{\omega_0} f_\omega^0 ds, \]  
(10')

where \( h_{x(\tau), s} = Y(\tau)Y^{-1}(s) \) is the Cauchy matrix; \( Y(\tau) \) is the fundamental matrix of the homogeneous system of equations

\[ \dot{y} = f_x y, \quad Y(0) = E. \]

Taking into account (10'), let us rewrite the system of constraints (12)-(14) in the compact form