The thermodynamic potential, magnetization, and spin contribution to the heat capacity of a biaxial antiferromagnet have been calculated over a wide range of temperatures and external magnetic fields for various limiting cases. The features of the thermodynamic quantities peculiar to a biaxial antiferromagnet have been studied.

As was noted in [1], experimental studies have been made of both antiferromagnets having a uniaxial magnetic anisotropy and the antiferromagnets CuCl₂·2H₂O [2] and CuSO₄ [3], which have a clearly defined biaxial magnetic anisotropy. Theoretical study of the thermodynamic properties of a biaxial antiferromagnet and comparison with the thermodynamic properties of a uniaxial antiferromagnet [4] are therefore necessary in order to determine the important differences in these properties which are associated with the biaxial anisotropy. Since there is much in common in the behavior of the two types of antiferromagnets, this approach should permit the clearest distinction between the experimental behavior of biaxial and uniaxial antiferromagnets.

Below, we calculate the magnetization and spin contribution to the heat capacity of a biaxial antiferromagnet over a wide range of magnetic fields for low temperatures, i.e., where the spin-wave theory is applicable.

We restrict the discussion to an antiferromagnet for which the magnet moments of the two equivalent sublattices are oriented along the OZ-axis (the axis of easy magnetization) in the absence of an external magnetic field. For such an antiferromagnet, the magnetic-anisotropy constants satisfy [1] \( \beta - \beta_\perp > 0 \), and \( \rho - \rho_\perp > 0 \) (an antiferromagnet of the \( A^Z \) type). The dipole-dipole interaction is neglected in the calculations of the thermodynamic properties.

The thermodynamic potential \( \Omega \) per unit volume of the antiferromagnet is given by

\[
\Omega = \sum_{j=1,2} \frac{T}{2\pi} \int d\kappa \ln \left( 1 - e^{-\frac{\epsilon_j}{T}} \right) = \frac{1}{24\pi^2} \sum_{j=1,2} \int \frac{k^2_j(\epsilon_j) d\epsilon_j dO_k}{\exp \frac{\epsilon_j}{T} - 1},
\]

where \( T \) is the temperature in energy units, \( \epsilon_j \) is the energy of the j-th type of spin wave, and \( dO_k \) is an element of solid angle in the direction of the wave vector \( \kappa \).

The magnetization \( m \) and the spin contribution \( C_s \) to the heat capacity are found from \( \Omega \) by means of the equations

\[
m = -\frac{\partial \Omega}{\partial H}, \quad C_s = -T \frac{\partial^2 \Omega}{\partial T^2}.
\]

Since we are interested here in finding those features of a biaxial antiferromagnet which are different from those of a uniaxial one, we limit the calculation of \( \Omega(T, H) \), \( m(T, H) \) and \( C_s(T, H) \) to those cases in which such features occur.
I. THE CASE $H || OZ$

1) The State $|| H_{zz}^{Z}$. If $H < \min (H_{\alpha 1}, H_{\alpha 2})$ (we are adopting the notation of [1]), the sublattice magnetic moments $M_{\alpha 1}$ and $M_{\alpha 2}$ are antiparallel and lie along the $OZ$-axis. The spin-wave energy $\varepsilon_j$ can be approximated in this case by

$$
\varepsilon_j = (\delta^2_{\omega} (ak)^2 + \delta^2 (HP + H_{zz}^{Z}) \pm [4 \delta_{\omega} (ak)^2 (\nu H)^2 + \nu (HP + H_{zz}^{Z})^2 - \nu^2 (H_{zz}^{Z} - H^2)(H_{zz}^{Z} - H^2)]^{1/2};
$$

$$
\eta^2_1 = (\nu M_{\omega}^\alpha)^2 \delta (a - a_{\perp})^2; \quad H_{I} = H_{\alpha 1} + H_{\alpha 2};
$$

$$
H_{\omega 1} = H_{\omega 1}^{\alpha} \frac{\beta - \delta}{\nu - \gamma}; \quad H_{\omega 2} = H_{\omega 2}^{\alpha} \frac{\beta - \delta}{\nu - \gamma}.
$$

We have adopted the usual values for $\delta$, $\gamma$, $\beta$, $\delta_{\omega}$, $\nu$, $\epsilon_{\perp}$, and $\rho_1$ [1,6].

Using Eqs. (1)-(3), we can find the $T$- and $H$-dependences of $\Omega$, $\varphi$, and $C_s$.

a) If $H \ll T \ll \nu H_{\alpha 1}$ ($v = 1, 2$), we have

$$
\Omega = -\frac{T}{(2\pi)^2 a} \sum_{\gamma = 1,2} \frac{(\nu H_{\alpha 1})}{(T/\gamma)} \left[ 1 - 4 \left( \frac{\nu H_{\alpha 1}}{T} \right)^2 \right] e^{-\frac{\nu H_{\alpha 1}}{T}},
$$

$$
\varphi = -\frac{8 \nu}{(2\pi)^2 a^2} \frac{HT}{\nu H_{\alpha 1}} \left( \frac{T}{\gamma} \right)^3 \sum_{\gamma = 1,2} \frac{(\nu H_{\alpha 1})}{(T/\gamma)} \left[ 1 - 4 \left( \frac{\nu H_{\alpha 1}}{T} \right)^2 \right] e^{-\frac{\nu H_{\alpha 1}}{T}},
$$

$$
C_s = -\frac{1}{2(2\pi)^2 a^2} \left( \frac{T}{\gamma} \right)^3 \sum_{\gamma = 1,2} \frac{\nu H_{\alpha 1}}{(T/\gamma)} \left[ 1 - 4 \left( \frac{\nu H_{\alpha 1}}{T} \right)^2 \right] e^{-\frac{\nu H_{\alpha 1}}{T}}.
$$

It is evident from (4) that these equations hold for $H = 0$ only for a biaxial antiferromagnet, since $(H_{\alpha 1})^2 = 0$ for a uniaxial antiferromagnet. Since spectrum (3) differs from that of a uniaxial antiferromagnet, the $T$- and $H$-dependences of $\Omega$ also differ in a similar case (when an internal radical appears).

b) If $(H_{\omega 1}/H_{\alpha 1}) \ll 1 \ll (T/\gamma)^2$, we have

$$
\Omega = -\frac{T}{a^2} \left( \frac{T}{\gamma} \right)^3 \left[ \frac{\pi^2}{45} + \frac{1}{6} \left( \frac{\nu H}{T} \right)^2 \right];
$$

$$
\varphi = -\frac{T}{a^2} \left( \frac{T}{\gamma} \right)^3 \left[ \frac{4 \pi^2}{15} + \frac{1}{3} \left( \frac{\nu H}{T} \right)^2 \right];
$$

Interestingly, the forms of the $T$- and $H$-dependences of $\Omega$, $\varphi$, and $C_s$ are the same as in the uniaxial case (see [4] for the case $\nu H_{\omega 1} \ll T \ll \theta_{\omega}$), although a transition cannot be made to the equations for a uniaxial antiferromagnet.

c) If $\nu [(H_{\gamma 1} - H) |H_{\alpha 1} - H_{\alpha 2}|]^{1/2} \ll \nu |H_{\alpha 1} - H_{\alpha 2}| \ll T \ll \theta_{\omega}$, we have

$$
\Omega = -\frac{T}{6a^2} \left( \frac{T}{\gamma} \right)^3 \left[ \frac{\pi^2}{45} \left( \frac{2H_{\alpha 1}}{|H_{\alpha 1} - H_{\alpha 2}|} \right) \right] - \frac{\pi^2}{40} \frac{H_{\alpha 1} - H_{\alpha 2}}{H_{\gamma 1} |H_{\alpha 1} - H_{\alpha 2}|} \left[ \frac{\nu (H_{\alpha 1} - H_{\alpha 2})^2}{2\pi T^2} \right];
$$

$$
\varphi = -\frac{T}{4a^2} \left( \frac{T}{\gamma} \right)^3 \left[ \frac{\nu H_{\alpha 1}}{(T/\gamma)} \right] \left[ \frac{T}{\nu H_{\gamma 1}} - \frac{T}{\nu (H_{\alpha 1} - H_{\alpha 2})} \right] - \frac{\nu (H_{\alpha 1} - H_{\alpha 2})^2}{2\pi T^2};
$$

$$
C_s = \frac{\nu^3}{5a^3} \left( \frac{T}{\gamma} \right)^3 \left[ \frac{2}{3} \left( \frac{2H_{\gamma 1}}{|H_{\gamma 1} - H_{\gamma 2}|} \right) + \frac{1}{4} \frac{H_{\gamma 1} - H_{\gamma 2}}{H_{\gamma 1} |H_{\gamma 1} - H_{\gamma 2}|} \right].
$$