THE COMPUTATION OF Q-SWITCHED LASER PULSES

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Two methods for computing the single pulse of three- and four-level lasers with instantaneous Q-switching are described. The first method makes it possible to calculate the energy, power, and duration of the pulse. The second method, based on the approximation of the population inversion by means of suitable functions, yields its shape. The analytical solutions are compared with those obtained on an electronic digital computer. The optimal losses and limits of applicability of the formulas derived in the case of noninstantaneous Q-switching are determined.

High-power optical pulses of short duration can be obtained from lasers operating in the so-called giant pulse mode [1]. The most efficient methods of obtaining powerful laser pulses are based on the regulation of the loss factor \( k_{\text{loss}} \) (or, as it is frequently expressed, on the switching of the quality factor \( Q = 2\pi n_{\text{mir}} k_{\text{loss}} \)) of the resonator. By increasing the loss artificially, one can bring about a substantial increase in the metastable level population for the duration of the pumping pulse. If the resonator loss is then reduced abruptly the laser generates a single pulse, or a train of pulses. The laser loss can be regulated by means of polarization shutters, rotating disks, total internal reflection prisms, nonlinear filters, and other devices [2].

In considering the radiation of Q-switched lasers, the authors of some theoretical papers [3–7] have assumed that the loss factor, \( k_{\text{loss}}(t) \), is a known function of time. Lasers with instantaneous Q-switching have been considered in [3, 8, 9]; paper [3] contains an expression for the peak power of the single pulse and approximate estimates of its duration. Paper [9] contains a method of computing laser pulses based on approximate analytical estimates. Papers [4–6], likewise, deal with the analytical computation of pulses. The radiation of a noninstantaneously Q-switched laser is considered in [7], where the principal features of the process are considered on the basis of numerical solutions. In the present paper we shall compute ruby and neodymium laser pulses with rapid Q-switching. The computations are based on an approximation procedure which allows one to determine the pulse shape. The limits of applicability and accuracy of the formulas derived are established. A generalization of the method for the case of noninstantaneous Q-switching will be carried out in a later paper.

The nonsteady-state radiation of Q-switched lasers can be computed on the basis of kinetic population equations and an equation for the radiation density,

\[
\frac{dy}{dt} = -2uy - Dy + G,
\]

\[\frac{du}{dt} = \frac{\nu_*}{1 + \frac{L}{l}} \left[ y - k_{\text{loss}}(t) \right] u + \epsilon.\]

Here \( u = B_\nu u^*/\Delta \nu \) is the probability of transitions between the operating levels stimulated by the frequency-integrated radiation of density \( u^* \); \( v^* \) and \( c \) are the speeds of light in the active medium and in a vacuum; \( l \) and \( L \) are the linear dimensions of the active medium and air gap (\( L + l \) is the interferometer base); \( y = (n_l - n_l)/n \) is the relative overpopulation of the metastable level; \( B \) is the Einstein coefficient; \( \Delta \nu \) is the width of the luminescence line; \( \nu = B_{\text{pum}}/\Delta \nu u^* \) is the limiting value of the gain; \( \epsilon \) is a coefficient representing the increase in \( u \) due to luminescence. The coefficients \( D \) and \( G \) describe the inversion increase due to the pumping radiation and the reduction of inversion through relaxation. They are given by

\[ D = \eta B_{\text{pum}} u_{\text{pum}} + p_{31}, \quad G = \eta B_{\text{pum}} u_{\text{pum}} - p_{31} \quad (p_{31} = 300 \text{ sec}^{-1}) \]

for ruby and

\[ D = 2 (\eta B_{\text{pum}} u_{\text{pum}} + p_{32} + p_{31}), \quad G = 2B_{\text{pum}} u_{\text{pum}} \]

\( (p_{32} = 800 \text{ sec}^{-1}, p_{31} = 600 \text{ sec}^{-1}) \)

for neodymium glass. Here \( B_{\text{pum}} u_{\text{pum}} \) is the transition probability due to the pumping light; \( p_{12} \) is the spontaneous transition probability \( 1 \rightarrow 2 \) (the levels are numbered from the ground level up); \( \eta \) is the quantum yield. The values of \( D \) and \( G \) for other four-level lasers (e.g., fluorite and \( \text{Sm}^{3+} \) or \( \text{U}^{3+} \)) are also easy to obtain.

The applicability of system (1), (2) as a means of describing a three-level laser was established in [10]. In the case
of four-level lasers it is assumed that the probability $P_{21}$ of particle "drainage" from the lowest operating (second) level of a neodymium laser is much larger than the probabilities of all the other processes. In the contrary case, neodymium lasers would have to be described by a more complex system of balance equations. On the other hand, the above assumption concerning the magnitude of $P_{21}$ is not germane to the computation of Q-switched neodymium lasers. Equations (1), (2) represent transient laser radiation; they describe radiation in a pulse mode as a special case only.

In order to investigate the effect of particle "drainage" from the lowest operating level of a neodymium laser on the character of its pulse radiation, we compared numerical solutions of a more complete system of balance equations ($P_{21} = 4 \times 10^6 \text{sec}^{-1}$, $8 \times 10^4 \text{sec}^{-1}$, et al.). It was found that the solutions coincided very closely (to within four significant figures), i.e., that the effect of "drainage" (the probability $P_{21}$) in these problems was not significant. Hence, four-level Q-switched lasers can also be described by means of equations (1), (2).

Moreover, the rejection of linear terms in equation (1) has practically no effect on the character of the solutions in question. Analysis of equations (1), (2) and their numerical solutions showed that the energy stored in the medium is radiated very rapidly, so that pumping and relaxation are practically ineffective in altering the population inversion. Hence, in our problem we can ignore the population increase due to spontaneous transitions, reducing our investigation of Q-switched lasers to the solution of the system

$$\frac{dy}{dt} = -2u y,$$

$$\frac{du}{dt} = v \left[ y - \frac{k_{\text{loss}}(t)}{L} \right] u, \quad v = \frac{\nu_0}{1 - \frac{L}{c}}.$$  \tag{4}

System (1), (2) was solved numerically on an electronic computer, while system (3), (4) was investigated analytically.

Estimation of the initial conditions in this problem does not require knowledge of the absolute pumping power. The Q-switched laser will be characterized by the inversion $y_0 = (n_0^2 - n^2)/n$ rather than by the pumping power. This inversion does not exceed the value $y_{\text{max}} = k_0^{-1} \nu_0$, where $k_0$ is the maximum loss factor of the system with a detuned resonator. The inversion $y_0$ is small if either the pumping power is small or the pumping lasts for a short time. This method of specifying the initial quantities makes the theory applicable regardless of pumping pulse shape. We note, incidentally, that for the purpose of achieving maximum power, switching generally takes place at the instant of maximum inversion, $y_0$, afforded by the pumping. The maximum value of the inversion at this instant is most readily determined by experiment.

The initial value of the generated power, $u_0$, can be estimated by computing the luminescence density from the formulas given in [11], with a correction for the solid angle of the generated radiation. Experiments have shown that the variation of $u_0$ within a fairly broad range (several orders of magnitude) has no marked effect on the basic generation characteristics.

Depending on the Q-switching rate, i.e., on the rate of change of $k_{\text{ loss}}(t)$, the solution $u(t)$ of systems (1)-(2), (3)-(4) is a curve with one or several maxima. In the present study we shall consider instantaneous switching,

$$k_{\text{loss}}(t) = \begin{cases} k_{\text{loss}}^{-1} & \text{for } t = 0 \\ k_{\text{loss}} = \text{const} & \text{for } t > 0, \end{cases}$$  \tag{5}

where $k_{\text{loss}} < k_0^{-1}$.

Instantaneous reduction of the resonator loss gives rise to a powerful single pulse followed by a train of low ordinary peaks [8] provided the continued pumping satisfies the self-excitation conditions. The subsequent low peaks are of no interest. They appear after a long time, or, if the loss begins to increase, do not appear at all. For this reason we shall confine our attention to the giant single pulse. In contrast to (1), (2), system (3), (4) describes only the giant pulse, since it does not include terms which contain pumping. The solutions for a neodymium laser (see Fig. 1) with instantaneous Q-switching were obtained on a "Minsk-2" computer. The solutions obtained were accurate to within 1%.

With rapid Q-switching, the powerful radiation pulse appears within nanoseconds. This is accompanied by an abrupt drop in the population inversion, though not all the way down to complete equalization of the populations (the radiation leaves the resonator rapidly, and only part of the energy stored in the medium is radiated).

Direct integration of (3), (4) makes it possible to determine the maximum radiation density inside the resonator [8, 9],

$$u_{\text{max}} = \frac{u_0 k_{\text{loss}}}{2} (z - 1 - \ln z).$$  \tag{6}