Interest in the present problem arose after the publication of the results of the experiments of Kramer [1-3]. In addition to the studies indicated in [4], the articles [5-8] are devoted to the question of the interaction of a flexible elastic surface with the boundary layer. In the present paper the problem of the interaction of an elastic surface with disturbances arising in the boundary layer is posed as in [4]. The approximate nature of the methods of solving the problem of the hydrodynamic stability of the laminar boundary layer leads to a difference in the final computational formulas even in the case when authors use the same Heisenberg-Tollmien-Schlichting-Lin scheme. Therefore, in what follows we present a comparison of the data on the stability of the boundary layer on a solid wall obtained by several authors with the calculations using the formulas, which are then generalized to the case of the elastic surface.

§1. Stability of laminar boundary layer on a rigid surface. When using the method of small perturbations [9], the problem reduces to finding the general solution of the Orr-Sommerfeld equation

\[(u - c)(f' - \alpha^2f) - u'f = -\frac{i}{\alpha R}(f'' - 2\alpha^2f' + \alpha^4f)\]  \hspace{1cm} (1.1)

written in dimensionless form in terms of the amplitude of the stream function \(f\) of the disturbed motion for the boundary conditions

\[f(0) = f'(0) = f'(1) + \alpha f(1) = 0,
\]

\[f(y) < M < \infty \text{ as } y \to \infty. \]  \hspace{1cm} (1.2)

In Eq. (1.1) the function \(u(y)\) is the distribution of the average velocity across the boundary layer, \(\alpha\) is the wave number which defines the length of the wave of the disturbing motion, \(c\) is the velocity of propagation of the disturbing motion, \(R\) is the Reynolds number based on the boundary layer thickness \(\delta\) and the velocity \(U\) at the edge of the boundary layer. These last two quantities are taken as the length and velocity scales, respectively. Using the known particular solutions of Eq. (1.1) and the homogeneous boundary conditions (1.2), we obtain [9] the characteristic equation

\[F(w)(1 + \Delta) = \frac{y_k u_k}{c} - 1, \quad \Delta = \frac{y_k u_k'}{c} - 1, \quad u(y_k) = u_k = c, \]  \hspace{1cm} (1.3)

In the first approximation we consider \(\Delta = 0\), which gives acceptable accuracy in calculating the critical Reynolds numbers. In principle it is possible to make the following approximation, determining the numerical values of the correction \(\Delta\) from the first approximation. In what follows we shall make use everywhere of the first approximation, which permits transforming (1.3) to the form

\[F^*(w) = 1 + z, \quad F^*(w) = \frac{F(w)}{F^*(w) - F(w) + iF^*(w)}, \]  \hspace{1cm} (1.4)

The complex equality (1.4) is equivalent to two real equalities

\[F^*(w) = 1 + z, \quad F^*(w) = \frac{F(w)}{F^*(w) - F(w) + iF^*(w)}, \]

\[w = y_k(u_k' x R)^{1/4}, \quad z = \frac{y_k u_k'}{u_k} - \frac{y_k u_k'}{u_k}, \]  \hspace{1cm} (1.5)

Here \(F^*(w)\) is the Lin function.

Using the scheme presented in [4], we can calculate with the aid of (1.5) the neutral stability curve for any laminar boundary layer velocity profile. The critical Reynolds number is defined by the formula (analogous to the known Lin formula)

\[R_* = 33u_0' (1 - c) (1.5 + 0.185 \ln c) / c. \]  \hspace{1cm} (1.6)

Figure 1 presents a comparison of the neutral stability curve for the Blasius profile calculated using Eqs. (1.5) (curve 1) with the analogous curves of Schlichting [10] (curve 2), Lin [9] (curve 3), Zaat [11] (curve 4), and also the experimental data of Schubauer and Skramstad [12] (points 5), Burns, Childs, Nicol, and Ross [13] (points 6), Hama [14] (points 7), and Wortman [15] (points 8). Figure 2 shows the variations of the critical Reynolds number based on the displacement thickness as a function of the Pohlhausen shape parameter when approximating the velocity profile by a sixth-power polynomial. Curve 1 is based on Eq. (1.6). The remaining relations correspond to the calculations of Schlichting and Ulrich [16] (curve 2), Zaat [11] (curve 3), Finston [17] (curve 4), Pretsch [18] (points 5), Teterzin [19] (points 6), Soprunenko [20] (points 7). The basis of the calculations of Schlichting and Ulrich, and those of Finston as well, who used the approximate equations of Lin, is the family of profiles defined by the Pohlhausen polynomial of sixth power. In Soprunenko's calculations the Howarth profiles were used, and in the calculations...
of Pretsch and Terervin use was made of the Hartree family of profiles. For his calculations Zaat used the family of profiles satisfying the asymptotic conditions as \( y \to \infty \). The conversion of the corresponding parameters was accomplished using the Schlichting method [16].

This comparison indicates that the use of the formulas mentioned above for the calculation of the stability leads to results which are in satisfactory agreement with the experiments and with the data of other authors. This situation is important since the technique used for calculating the stability of the laminar boundary layer in the case of a rigid surface is extended below to the case of elastic surfaces.

§2. Stability of laminar boundary layer on a surface which is compliant in the normal direction. The boundary conditions on the compliant surface in this case are posed just as in studying the corresponding problem with application to the Poiseuille flow [4]. Use is made of the following connection between the surface deformations and the normal stress of the pulsating pressure on the wall

\[
y^\circ = k^\circ p_{yy} e^{i\theta_1}, \tag{2.1}
\]

Here \( k^\circ \) is a constant which depends for a given disturbance on the coating properties, \( p_{yy} \) is the variable component of the pressure on the surface of the immersed body, and \( \theta_1 \) is the phase shift between the oscillations of the surface stress and the corresponding forced oscillations of the wall. The symbol \( \circ \) denotes dimensional quantities. The normal stress acting in the viscous fluid on an area perpendicular to the axis of ordinates is expressed by the formula

\[
p_{yy} = -p + 2\mu \frac{\partial v_y}{\partial y}. \tag{2.2}
\]

If the wall does not deform in the tangential plane, the derivative \( \partial v_x / \partial x = 0 \) for \( y^\circ = 0 \), and consequently, in view of the equation of continuity, which is assumed valid right up to the fluid boundary, \( \partial v_x / \partial y^\circ = 0 \) for \( y^\circ = 0 \). Therefore we shall understand the quantity \( p_{yy} \) to be the static pressure \( p \), including the minus sign in the phase shift. As a result, the boundary conditions at the wall are written as [4]

\[
(k c u_0^\circ - e^{i\theta_1}) f (0) + k c \frac{f' (0)}{i\alpha R} = f' (0) = 0, \quad \left( k = \frac{\nu \rho U^2}{k^\circ} \right). \tag{2.2}
\]

We take the boundary conditions in the stream to be the same as in the case of the rigid surface (1.2). The dimensionless quantity \( k \) plays the role of a similarity criterion in modeling the conditions of operation of the elastic coating.

In spite of its simplicity, the relation (2.1) includes all the particular cases considered in the studies mentioned in [4]. Thus, for example, let the motion of the surface be described by the equation

\[
a_1 \frac{\partial y_1}{\partial t} + a_2 \frac{\partial y_1}{\partial x} + a_3 y_1 + a_4 \frac{\partial y_1}{\partial y} = p(x, t),
\]

\[
(p(x, t) = p_0 e^{i(x - ct)}).
\]

We seek the forced oscillations of the wall in the form \( y_1 = y_{10} \exp [i\alpha (x - ct)] \); for determining \( k \) and \( \theta_1 \) we obtain

\[
k c e^{i\theta_1} = (a_2 - a_3^2) a_1 - i a_3 a_2 - a_4^2)^{-1}.
\]

The boundary conditions (2.2) together with the last two of conditions (1.2) lead to the complex characteristic equation

\[
G(w, kc, \theta_1) = 1 + z,
\]

which is equivalent to two real equations

\[
G_r = 1 + z_r, \quad G_i = z_i. \tag{2.3}
\]

In equalities (2.3) the quantities \( z_r, z_i \) are defined just as in Eqs. (1.5);

\[
G_r = \frac{1 - F_r (w) + A}{(1 - F_r + A) + (B - F_r)^2}, \quad A = \frac{k c u_0^\circ (\cos \theta_1 - k c u_0^\circ)}{1 - 2 k c u_0^\circ \cos \theta_1 + k c u_0^\circ},
\]

\[
G_i = \frac{F_i (w) - B}{(1 - F_r + A) + (B - F_r)^2}, \quad B = \frac{k c u_0^\circ \sin \theta_1}{1 - 2 k c u_0^\circ \cos \theta_1 + k c u_0^\circ}.
\]

Here \( F(w) = F_r (w) + i F_i (w) \) is the Tietjens function.

The system of equations (2.3) permits the calculation, using the schemes presented in [4], of the curve of neutral stability for any velocity profile for given values of \( k, \theta_1 \), and also permits the direct determination of the critical Reynolds number. Figure 3 shows the results of the calculations of the variation of the critical \( R \) as a function of the parameter \( \theta_1 \) (curve 1) made for the Blasius profile with \( k = 0.1 \). The dashed line 2 corresponds to the critical \( R \) for the rigid plate. The relations \( R^* = R^* (k, \lambda) \) in the case \( \theta_1 = 60^\circ \) are shown in Fig. 4.

It is of interest to consider, on the basis of the results obtained, the direction of the energy flux for interaction of pressure pulsations in the stream with