The problem of quantum scattering for a system of $n$ particles in the framework of the Friedrichs model is considered. The structure of the singularities of the wave operators is established, and their completeness is proved on the basis of this. The behavior of the scattering matrix for large energies is studied.

The basic problem of mathematical scattering theory is the development of prescriptions for constructing complete systems of eigenfunctions for the continuous spectrum of self-adjoint operators of special classes and the justification of these prescriptions. For the energy operator $\hat{H}$ of a system of $n$ quantum-mechanical particles such prescriptions are known from physical considerations, and the existing methods of justification are based on reducing the problem to integral equations for the kernel of the resolvent of the operator $\hat{H}$. Such equations are constructed directly and have a relatively simple character for $n=2$; they have been developed in detail, and there are many versions of the theory of these equations. For systems with a greater number of particles the very construction of equations which can be satisfactorily included in the theory of equations with a completely continuous operator is quite an onerous procedure [1-4]. For $n>3$ the investigation of such equations has so far been fraught with difficulties. Recently V. S. Buslaev and the author suggested a new approach to the justification of scattering theory for a system of several particles. In the present paper we apply this approach to study an operator which generalizes in a natural way the Friedrichs model of an operator for a system of three particles [5].

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1.

In this section we describe the Friedrichs model for the energy operator of a system of $n$ particles and introduce the basic objects of scattering theory connected with this model.

The self-adjoint operator $\hat{H}$ of the $n$-particle Friedrichs model acts in the space
\[ H = L^2(\mathbb{R}^n \rightarrow N) \] of functions on \( \mathbb{R}^n \) with values in the Hilbert space \( N \). The action of \( H \) is given by the formula \( H = H_0 + \sum_{k=1}^N V_k \), where \( (H_0 f)(x) = -\sum_{k=1}^N f_k(x) \) \( f(x) \), \( f_k(x) \) is the derivative of order \( k \) with respect to \( x_k \).

\[ (V_k f)(x) = \int \nu_k(x_k, x'_k) \delta(x_1, \ldots, x_{k-1}, x_k', x_{k+1}, \ldots, x_n) \, dx_k' \]

The functions \( \nu_k \), which are defined on \( \mathbb{R}^n \) and assume values in the set of completely continuous operators on \( N \), are assumed to have derivatives of all orders which decrease faster than any power of \( (1 + |x_k| + |x'_k|)^{-t} \). Such functions we henceforth call smooth, decreasing functions.

In a system of \( n \) particles subsystems can be distinguished which consist of a smaller number of particles. In the Friedrichs model to these there correspond subsets \( \alpha \) of the set \( \{1, 2, \ldots, n\} \) and operators \( H_\alpha = H_0 + \sum_{k \in \alpha} V_k \). We write a vector \( x \in \mathbb{R}^n \) in the form \( x = (x_\alpha, x_\bar{\alpha}) \), where \( \bar{\alpha} = \{1, 2, \ldots, n\} \setminus \alpha \) and \( x_\alpha \) is a vector with components \( x_k, k \in \alpha \). The operator \( H_\alpha \) is hereby written in the form \( H_\alpha \otimes H_0 \). The operator \( H_\alpha \) acts on functions of \( |\alpha| \) variables (\( |\alpha| \) is the number of elements in \( \alpha \)) and represents the Friedrichs model of the energy operator for a system of \( |\alpha| \) particles. Let \( \varepsilon \) be an eigenvalue of the operator \( H_\alpha \), \( \psi(\alpha) \) be an eigenfunction of \( H_\alpha \) with eigenvalue \( \varepsilon \), let \( f(\alpha) \) be an arbitrary square-summable (scalar) function, and let \( P_{\alpha, \varepsilon} \) be the projector in \( \mathcal{H} \) onto the subspace generated by functions of the form \( f(\alpha) \psi(\alpha) \). We consider the following limits (assuming that they exist):

\[ W_{\alpha, \varepsilon}^{(\alpha)} = \lim_{t \to -\infty} \exp(iHt) \exp(-iH_\alpha^{(\alpha)}) P_{\alpha, \varepsilon} \]

(1.1)

(for \( \alpha \neq \emptyset \), \( P_{\emptyset} = I \), and the index \( \varepsilon \) is omitted). The definition gives the following properties of the operators \( W_{\alpha, \varepsilon}^{(\alpha)} :

1. \( W_{\alpha, \varepsilon}^{(\alpha)} \) is an isometric operator on \( P_{\alpha, \varepsilon} \mathcal{H} \).
2. The ranges of the operators \( W_{\alpha_1, \varepsilon_1}^{(\alpha)} \) and \( W_{\alpha_2, \varepsilon_2}^{(\alpha)} \) are orthogonal if \( \alpha_1 \neq \alpha_2 \) or \( \varepsilon_1 \neq \varepsilon_2 \).
3. \( H W_{\alpha, \varepsilon}^{(\alpha)} = W_{\alpha, \varepsilon}^{(\alpha)} H_\alpha \).

The subspace \( P_{\alpha, \varepsilon} \mathcal{H} \) is equivalent in a natural way to the space \( \mathcal{H}_{\alpha, \varepsilon} = L^2(\mathbb{R}^{|\alpha|} \rightarrow N_{\alpha, \varepsilon}) \), where \( N_{\alpha, \varepsilon} \) is the eigensubspace of the operator \( H_\alpha \) corresponding to the eigenvalue \( \varepsilon \). The space \( \mathcal{H}_{\alpha, \varepsilon} \) is called a scattering channel. The operator \( W_{\alpha, \varepsilon}^{(\alpha)} \) can thus also be considered on \( \mathcal{H}_{\alpha, \varepsilon} \). We denote by \( \mathcal{H} \) the orthogonal sum \( \mathcal{H} \oplus (\bigoplus_{\alpha, \varepsilon} \mathcal{H}_{\alpha, \varepsilon}) \) and by \( W_{\alpha}^{(\alpha)} \) the operator from \( \mathcal{H} \) to \( \mathcal{H} \), the components of which are the operators \( W_{\alpha_1, \varepsilon_1}^{(\alpha_1)} \) and \( W_{\alpha_2, \varepsilon_2}^{(\alpha_2)} \). We denote also by \( \mathcal{H}_0 \) the diagonal operator in \( \mathcal{H} \); its diagonal elements are the operators \( H_0 \) and \( H_\alpha^{(\alpha)} + \varepsilon \). Then (1.2) is equivalent to the following:

1. \( W_{\alpha}^{(\alpha)} \) is an isometric operator;
2. \( H W_{\alpha}^{(\alpha)} = W_{\alpha}^{(\alpha)} \mathcal{H}_0 \).

(1.3)