A GENERALIZATION OF THE METHOD OF ENCLOSING SURFACES IN CLASSICAL RADIATION THEORY

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The method of enclosing surfaces in classical radiation theory is generalized to the case of a charged particle moving in a transparent isotropic dispersive medium. The synchrotron and ondulator radiation problems are discussed, and expressions are derived and analyzed for the angular-spectral distribution of the radiation power in these cases.

The development of precise analytical methods is of paramount importance in the mathematical interpretation of physical phenomena. For example, in the interpretation of the Cerenkov effect following the study [1], in which asymptotic expressions were used for the Hankel functions, the exact field expressions were invoked in [2] to obtain the fundamental theoretical relation describing the radiation intensity and spectral distribution. Rigorous methods have been applied in several studies of synchrotron radiation [3-5]. The method of Lorentz transforms was elaborated in [3, 4] to find the angular-spectral distribution of the radiation intensity of a charge in external fields.

The distinctive feature of the method of [5] is the fact that the radiation intensity from an electric charge moving in a circular or helical path is calculated in terms of a cylindrical surface that encloses the path of the radiator, but is not necessarily situated at an infinite distance from it.

We now seek to generalize the rigorous method of [5] to the case of radiation from a charged particle moving in a prescribed path in a transparent isotropic dispersive medium. The radiation from charged particles moving according to a prescribed law in such a medium has highly specific characteristics associated with the possible occurrence, under definite conditions, of Cerenkov radiation, which has extensive applications in high-energy particle physics [6-8]. The application of radiators of finite dimensions in Cerenkov counters and the corroboration of the feasibility of using ondulator radiation for the identification and energy measurement of ultrafast charged particles impart a special timeliness to the above-stated problem.

The delayed vector potential \( A_{\text{ret}}(\mathbf{r}, t) \) and delayed scalar potential \( \varphi_{\text{ret}}(\mathbf{r}, t) \) of the electromagnetic field of a moving charged particle are found by solving the appropriate differential equations and have the form

\[
\begin{align*}
(A_{\text{ret}}(\mathbf{r}, t)) &= e^{\frac{1}{c^2} \int_{-\infty}^{t} \frac{dz'}{\sqrt{j^2 - \mu^2}}} 
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \frac{\varphi_{\text{ret}}(t')}{c} \, dz' \, dx' \, dy', \\
\varphi_{\text{ret}}(\mathbf{r}, t) &= \frac{\varphi_{\text{ret}}(t')}{c} \, dz' \, dx' \, dy', \\
&\times G(\kappa, w) \exp \left[ i \left[ R \cos \varphi - \Psi \right] - x \cos \varphi \xi' - y \sin \varphi \eta' - z \left( z - z_{\text{ret}}(t') \right) - w \left( t - t' \right) \right],
\end{align*}
\]

where the Green functions \( G(\kappa, w) = \left( \kappa^2 - \frac{n^2(w)}{c^2} \right)^{-1} \), \( \kappa_x = \kappa \cos \varphi \), \( \kappa_y = \kappa \sin \varphi \), \( \mu(w) \) and \( \nu(w) \) are the dielectric parameter and magnetic permeability of the transparent medium, \( c \) is the speed of light in vacuum, \( \mathbf{r}_{\text{ret}}(t) = x_{\text{ret}}(t)\mathbf{i} + y_{\text{ret}}(t)\mathbf{j} + z_{\text{ret}}(t)\mathbf{k} \) is the law governing the motion of the particle, and \( e \) is its charge.


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The radiation from an electric charge moving in a circular path in a transparent isotropic dispersive medium has been treated in [9, 10] by the wave-zone method. The radiation from an electric charge moving in a helical path in a medium has been analyzed in [11, 12] on the basis of the method of the Lorentz self-action force. We now solve this problem on the basis of the rigorous method of Sokolov and co-workers [5].

The law describing the motion of the charge has the form
\[
x_p = -r_0 \cos \tilde{\omega} t, \quad y_p = r_0 \sin \tilde{\omega} t, \quad z_p = \nu_1 t,
\]
where
\[
r_0 = v_1 \omega^{-1}, \quad \tilde{\omega} = \frac{c \mathcal{E} B_{\text{ext}}}{\mathcal{E}}, \quad \mathcal{E} = c \sqrt{v^2 + m_0^2 c^2}, \quad B_{\text{ext}}
\]
is the induction vector of an external magnetic field directed along the z axis. Using the expansion of a plane wave in Bessel functions, we write Eq. (1), subject to conditions (2), in the form
\[
\begin{align*}
&\left( A^{\text{ext}}(r, t) \right. \\
&\left. \nu^{\text{ext}}(r, t) \right) = \frac{e}{2 \pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d \omega}{\omega^2} \left( \frac{n(\omega)}{c} \left( \nu_1 \sin (\omega t') + \nu_1 \cos (\omega t') \right) \nu_1 \omega^{-1}(\omega) \\
&+ \alpha \nu_1 \omega^{-1}(\omega) \right) \int_{-\infty}^{\infty} d\kappa_2 \exp \left( i \kappa_2 \left( z - \nu_1 t' \right) - \omega (t - t') \right)
\end{align*}
\]
\[
\times \sum_{m = -\infty}^{\infty} (-1)^m e^{-i m \omega t} \left( \frac{\pi}{2} \eta \left( \frac{n(\omega)}{\omega} \right) - \nu_1 \alpha \right) J_m(\gamma_1 r_0) H_m(\gamma_1 R),
\]
where
\[
\gamma_1 = \sqrt{\frac{n^2(\omega)}{c^2} - \kappa_2^2}, \quad \gamma_2 = \sqrt{\kappa_2^2 - \frac{n^2(\omega)}{c^2}}, \quad \eta(\omega) = \begin{cases} 1, & \omega > 0 \\ 0, & \omega < 0 \end{cases}
\]
\[
H_m(\gamma_1 R) = i \frac{\omega}{\gamma_1} J_m(\gamma_1 R) - N_m(\gamma_1 R), \quad J_m \quad \text{and} \quad N_m \quad \text{are integer-order Bessel functions of the first and second kind,} \quad I_m \quad \text{and} \quad K_m \quad \text{are, respectively, the Bessel function of an imaginary argument and the Macdonald function,} \quad R > r_0. \quad \text{We calculate the intensity of radiation from the charged particle across a cylindrical surface having the z axis as its axis and radius R and enclosing the path of the particle:}
\]
\[
P = \frac{eR}{\pi} \lim_{R \to \infty} \frac{1}{2\pi} \int_0^{2\pi} dt \int_0^\infty dz \int_0^\infty \frac{da}{\theta} (E_n H_z - E_z H_n).
\]
The field components must be calculated by means of the potentials (4) according to the standard equations. For \( \frac{n^2(\omega) \omega^2}{c^2} < \kappa_2^2 \) the integral (5) is equal to zero. For \( \frac{n^2(\omega) \omega^2}{c^2} > \kappa_1^2 \) the integration must be carried out with regard for: the theorem that the limit of the product of a finite number of variables is equal to the product of their limits; the expression
\[
J_m(\gamma_1 R) N_m(\gamma_1 R) - J_m(\gamma_1 R) N_m(\gamma_1 R) = \frac{2}{\pi \gamma_1 R} ;
\]
and the recursion formulas 8.471 and 8.477 given in [13] for the Bessel functions. We finally obtain for the radiation power:
\[
P = \frac{e^2}{c^3} \int_0^\infty d\omega \int_0^\infty d\theta \sin \theta \int_0^\infty \sum_{m = -\infty}^{\infty} \delta (\omega - n\omega_0) \sin \omega \Theta - m \tilde{\omega}
\]
\[
\times \left[ \left( \nu_1 - c' \cos \Theta \right)^2 \frac{1}{\sin^2 \Theta} J_m(q) + \nu_1^2 J_m^2(q) \right],
\]
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