CHOICE OF THE PARAMETERS OF A MAGNETIC UNDULATOR

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The total output power of undulator radiation is investigated as a function of the constructional parameters of the undulator and its supply current. Calculations are presented of the magnetic field in the undulator, and the optimum undulator dimensions for maximum magnetic field intensity per unit length of the system are calculated. It is shown that the total radiated power of the charged particles depends on the undulator periodicity.

The possibility of using magnetic undulators in electron synchrotrons and storage devices [1, 2] to obtain high-power electromagnetic radiation in the vacuum ultraviolet and x-ray regions of the spectrum has recently been widely discussed. It opens up considerable possibilities for investigations in solid-state spectroscopy, in molecular physics, in high-energy physics, photochemistry, and biology. The theory of undulator radiation has been considered in [3-5], where the dependence of the spectral-angular and polarization properties of the radiation on the periodicity and value of the magnetic field in the undulator was pointed out.

In this paper we investigate the dependence of the total undulator radiation power on the distance between the poles of the magnets $2h$, the length of the periodicity elements of the undulator $2p$, and the supply current $I$ (see the notation in Fig. 1).

We will consider an ion-free undulator consisting of single-winding solenoids of rectangular cross section. The low ohmic resistance ($\sim 10^{-3} \Omega$) and inductance ($\sim 10^{-5} \text{ H}$) of this system enables us to supply the undulator with short ($\sim 10^{-3} \text{ sec}$) high-power current pulses ($\sim 30 \text{ kA}$) which makes it convenient to use in charged-particle accelerators. This system also enables us to calculate in an analytical form on a computer the trajectories of motion of the electrons in the undulator, and, consequently, the characteristics of the undulator radiation in any specific arrangement.

Taking into account the radial dimensions of the electron beam and the region occupied by it when the electrons move through the undulator in a radial direction, it is easy to satisfy the required radial uniformity of the magnetic field in the sections of a system of simple variations of the width $2\alpha$ of the rectangular solenoids.

Numerical calculations on a computer enabled us to obtain the optimum dimensions of the current loop of the solenoid of one section in order to obtain the maximum magnetic-field strength per unit length of the undulator, and enabled us to calculate the distribution of the field components in its volume.

To ensure that the field distribution in the system is independent of the properties of the supply source, we took into account in the calculations only series connection of the current windings, which in addition gets rid of various instabilities in the field distribution that occurs in the case of parallel connection. We also assumed the section of the conductor with the current to be infinitely small. For a

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**Fig. 1.** Sketch illustrating the notation used to calculate the magnetic-field distribution in the undulator.
medium with a relative magnetic permeability $\mu_r = 1$ (there are no ferromagnetic materials present), using the Biot–Savart law we can write for a single magnetic section

$$dH_x = \frac{I}{r} [(z - z_i) dy_i],$$
$$dH_y = \frac{I}{r} [(- z + z_i) dx_i],$$
$$dH_z = \frac{I}{r} [(y - y_i) dx_i - (x - x_i) dy_i],$$

where $r = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}$; $x, y, z$ are the instantaneous coordinates in the magnetic field distribution, and $x_i, y_i, z_i$ are the coordinates of the element $dl$ in the current-loop circuit.

After integrating Eqs. (1) over the loop contour we obtain the following expressions for the magnetic-field components in the undulator:

$$H_{xln} = \sum_{n=1}^{N} \sum_{i=1}^{2} dH_{xin} = (-1)^n \frac{I}{r} \left[ \frac{y_2 - y_1}{r_{22}} \frac{y_1 - y_2}{r_{11}} \right],$$
$$H_{yln} = \sum_{n=1}^{N} \sum_{i=1}^{2} dH_{yln} = (-1)^n \frac{I}{r} \left[ \frac{x_2 - x_1}{r_{22}} \frac{x_1 - x_2}{r_{11}} \right],$$
$$H_{zln} = \sum_{n=1}^{N} \sum_{i=1}^{2} dH_{zln} = (-1)^n \frac{I}{r} \left[ \frac{z_2 - z_1}{r_{22}} \frac{z_1 - z_2}{r_{11}} \right].$$

where $C_n$ is the distance from the origin of coordinates to the projection of the center of the $n$-th loop on the $x$ axis, $C_0$ is the distance from the origin of coordinates to the center of the first loop, $C_n = C_0 + (n-1)p$; $p$ is the distance between the centers of neighboring loops, $(-1)^n$ is a factor that takes into account the periodicity of the magnetic-field direction, $n = 1, 2, 3...$ is the order number of the coils with current measured from the origin of coordinates, and $i$ is an index that represents the position of a coil in a plane parallel to the $xy$ plane ($i = 1$ below the undulator axis and $i = 2$ above the undulator axis).

Hence, we can obtain the magnetic field at any point in the undulator by summing with respect to $i$ and $n$ in Eqs. (2)

$$H_q = \sum_{i=1}^{2} \sum_{n=1}^{N} C_{qin} = \sum_{i=1}^{2} \sum_{n=1}^{N} (-1)^n G_{qin},$$

where $N$ is the number of sections in the undulator, and $G_{qin} (q=x, y, z)$ is a geometrical factor which represents the expressions in the square brackets in Eqs. (2).

As is well known [6], the total power radiated by an electron when it moves in a circle of radius $R$ in a magnetic field $H$ is

$$W_0 = \frac{2 \epsilon c^2}{3} \gamma^4 \beta^2 = \frac{2 \epsilon c^4}{3 E_0} \gamma^2 \beta^2 H^2,$$

where $\gamma = E/E_0$, $E$ is the electron energy, $E_0 = mc^2$, $\beta = V/c$, $V$ is the electron velocity, $c$ is the velocity of light, and $e$ is the electron charge. An electron passing through an undulator of length $L = Np$ radiates a total energy given by

$$E_L = \int_0^L W_0 \frac{1}{\beta c} dx \approx N \int_{(n-1)p}^{np} W_0 \frac{1}{\beta c} dx \approx$$

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