In thermally stressed nuclear installations, solid-phase inactive elements of the reactor (the moderator, shells of fuel elements and of fuel channels, supporting elements of the construction, etc.) can operate in a regime of large thermal charges, being formed both by convective heat exchange from the fuel elements and the coolant, and also from the considerable heat release due to the γ-radiation. For small geometric dimensions and large energy release the characteristic times of heating of the inactive elements can be measured in seconds, i.e., they will be comparable with the decay constant of nuclei of emitters of delayed neutrons. The inactive elements made a considerable contribution to the neutron multiplication constant in the reactor core, often positive in sign.

Under these conditions the effect of the inactive elements on the dynamic processes in the reactor can prove to be so significant that they can cause an instability in the stationary regime of operation of the nuclear installation.

We note that the rates of thermal processes in the inactive elements of thermally stressed reactors, as a rule, are considerably less than the characteristic rates of variation of processes in fuel elements and the coolant. Hence, the calculation of the dynamic effect of the inactive elements can be taken as a system of differential equations that describes the relatively slow processes, in which the effects of the nuclear fuel and the coolant are taken into account in the form of the instantaneous power contribution to the reactivity. A theoretical basis for the effect of such slow processes can be found in [1].

It is evident that the complete system of differential equations that describes the dynamics of the slow processes in the reactor should include the distributed equations of thermal conductivity for each inactive element, and, perhaps with some difficulty, should yield a qualitative analysis of the stability. The problem is considerably simplified if the partial differential equations are replaced by the averaged ordinary differential equations; in this case, the distributivity can be represented in the form of a time delay in deviations of the characteristic elements that form the total reactivity [2].

Thus, the mathematical model that describes the slow processes in the reactor can be represented in the following form:

\[
\frac{dN(t)}{dt} = \frac{\delta k(t)}{l} - \frac{\beta}{l} N(t) + \sum_{i=1}^{6} \lambda_i C_i(t);
\]

\[
\frac{dC_i(t)}{dt} = \frac{\beta_i}{l} N(t) - \lambda_i C_i(t), \quad i = 1, 2, \ldots, 6;
\]

\[
\delta k(t) = \varepsilon_0 [N(t) - N_0] + \sum_{k=1}^{M} \varepsilon_k [T_k(t) - T_{k0}];
\]

\[
m_{hk} \frac{dT_k(t)}{dt} = m_h \left[ A_h N(t) + B_h \int_0^\infty N(t - u) f_h(u) du \right] - K_h [T_h(t) - T_{h0}].
\]
Here (1) and (2) are the equations of neutron kinetics, written in the usual form [3]*; Eq. (3) is the equation for the reactivity, in which $\varepsilon_0$ and $\varepsilon_k$ are the corresponding coefficients of reactivity. The heat-balance equation (4) in the k-th inactive element of the reactor takes account of the convective heat exchange and the heat release in the element, which in the general case is the sum of the energy of the neutron moderation, proportional to the instantaneous power of the reactor, and the energy of the absorbing $\gamma$-radiation of the reactor, which is separable into instantaneous and delayed components [4]. The function $f_k(u)$ characterizes the decrease in power of the delayed $\gamma$-radiation from the fission that occurs at the moment $u = 0$; the function $f_k(u)$ normalized to a single fission can be written in the form $f_k(u) = \sum_{j=1}^{m} g_{kj} e^{-\gamma_k j u}$. We have further that $T_k(t)$ is the mean temperature of the k-th element; $m_k$, $c_k$, and $K_k$ are the mass, specific heat, and coefficient of heat transfer of the k-th element, respectively; $A_k$, $B_k$, $g_{kj}$, and $\gamma_k$ are positive constants; $\tau_k$ is a delay constant that characterizes the thermal inertia in the k-th element, together with the time constant in Eq. (4). The temperature of the medium that exchanges heat with the inactive element is assumed constant ($T_{kin} = const$). The subscript $^0$ in Eq. (3) corresponds to the stationary value of the variable.

We introduce the dimensionless deviations of the variables

$$n(t) = \frac{N(t) - N_0}{N_0}; \quad \bar{n}_i(t) = \frac{C_i(t) - C_{i0}}{C_{i0}},$$

for $i = 1, 2, \ldots, 6; \quad \bar{n}_k(t) = \frac{T_k(t) - T_{k0}}{T_{k0}}, \quad k = 1, 2, \ldots, M$.

and the notation $\mu_0 = \varepsilon_0 N_0^0; \quad \mu_k = \varepsilon_k T_k^0; \quad \alpha_k = A_k N_0^0/C_k T_k^0; \quad \beta_k = B_k N_0/C_k T_k^0; \quad \gamma_k = K_k/c_k m_k; \quad k = 1, 2, \ldots, M$. We also introduce the new variable $\xi_{kj} = b_k g_{kj} \int_0^t n(u) e^{-\gamma_k j (t-u)} du$. As a result, the system (1)-(4) acquires the form:

$$\frac{d\bar{n}_i}{dt} = -\sum_{j=1}^{6} \bar{\beta}_i \bar{n}_j (n - \bar{n}_j) + \sum_{k=1}^{M} \mu_k \bar{n}_k (t - \tau_k),$$

$$\frac{d\bar{n}_k}{dt} = \lambda_k (n - \bar{n}_k), \quad i = 1, 2, \ldots, 6;$$

$$\frac{d\bar{\theta}_h}{dt} = \alpha_h n - \beta_h \bar{\theta}_h + \sum_{j=1}^{m} \bar{\xi}_{kj};$$

$$\frac{d\bar{\xi}_{kj}}{dt} = b_k g_{kj} \int_0^t n(u) e^{-\gamma_k j (t-u)} du.$$

We investigate the stability of the trivial solution of the system (5)-(8) using the second method of Lyapunov, generalized to a system with time delay [5]. With this goal in mind, we consider the functional

$$V = \sum_{i=1}^{6} \sum_{j=1}^{M} \bar{\beta}_i \bar{n}_j \left[ n_i - \bar{n}_j \right] + \sum_{k=1}^{M} \left[ \bar{\theta}_k^2 + 2 R_k \int_{t - \tau_k}^{t} \bar{\theta}_k (\psi) d\psi \right] + \sum_{h=1}^{m} \sum_{j=1}^{M} \bar{\xi}_{kj}^2,$$

where $R_k$ is some positive constant.

Its total time derivative, calculated from the equations of system (5)-(8), has the form

$$\frac{dV}{dt} = -\sum_{i=1}^{6} \bar{\beta}_i \frac{(n - n_i)^2}{(n + \bar{n}_i + (n - \bar{n}_i))} \sum_{k=1}^{M} U_k [n, \theta_k, (t - \tau_k)] - \sum_{h=1}^{m} \sum_{j=1}^{M} W_{kj} [n, \theta_h, \xi_{kj}],$$

where

$$U_k [n, \theta_k, (t - \tau_k)] = -\frac{\eta_0}{2 M} n^2 - 2 a_h n \theta_k - \mu_k n \theta_k (t - \tau_k) + (d_h - R_k) \theta_k^2 + 2 R_k \bar{\theta}_k (t - \tau_k);$$

$$W_{kj} [n, \theta_h, \xi_{kj}] = -\frac{\eta_0}{2 M m} n^2 - 2 b_k g_{kj} n \xi_{kj} - 2 b_k g_{kj} \bar{\theta}_h \xi_{kj} + \frac{d_h - R_k}{m} \theta_k^2 + 2 \bar{\theta}_k \xi_{kj}^2;$$

*In the dynamics of "slow" processes the parameter $1/\beta$ is also small and unimportant; however, it is retained for convenience for the subsequent construction of a Lyapunov functional. Such a partial taking into account of insignificantly small parameters is based on [1].