of the structure of the ray field begins which grows stronger with increasing \( t \). For \( t > 0 \) the rays recede rapidly from the boundary, and to the right of the point \( t = +1 \) a shadow zone is formed with an approximate boundary which is the limit ray* shown in Figs. 1 and 2 by the broken line. Comparison of these figures with the shadow figures (see [1]) shows that the regions of maximum value of the wave field (the darkened regions) coincide with the regions occupied by the rays.

In the case of a flat point of a concave boundary, the behavior of the family of rays associated with the first and second whispering gallery waves is shown in Figs. 3 and 4, respectively. The structure characteristic of whispering gallery waves is preserved up to \( t \approx -2 \). On the interval from \(-2\) to \(+2\), the rays recede rapidly from the boundary, and, beginning at \( t \approx +3 \), they are again concentrated near the boundary and generate a rather complicated ray pattern. We note that it is just in this region (i.e., for \( t \approx +3 \)) in the shadow figures (see [2]) that the interference pattern occurs which is occasioned by the excitation beyond the flat point \( t = 0 \) of the boundary of a large number of whispering gallery waves. Just as in the case of an inflection point of the boundary, the regions of maximum value of the wave field in the shadow figures [2] coincide with the regions occupied by rays.

Thus, there is a clear connection between the qualitative behavior of the wave field and the ray dynamics in a neighborhood of the flat points considered of the boundary.

**LITERATURE CITED**


*The limit ray is the ray tangent to the boundary at the inflection point \( t = 0 \). Its equation, as follows from formula (10,a), has the form \( x = (1/6)t^3 \).

**FINITE EXPRESSIONS FOR THE CHARACTERISTIC MATRICES OF WEAKLY CURVED ELASTIC LAYERS**

**L. A. Molotkov**

The propagation of two-dimensional waves in cylindrical and spherical, homogeneous, elastic layers is investigated. For these layers finite formulas are found for the characteristic matrices. Comparison of these matrices and use of asymptotic representations for the Hankel functions make it possible to derive expressions in the case of weakly curved elastic layers.

The expressions obtained correspond to analogous formulas in the form of matrix series.

This work is devoted to the description of the two-dimensional propagation of waves in a weakly curved, homogeneous, isotropic, elastic layer in which the radius of curvature is many times larger than the dominant...
wave. For this layer an expression is derived for the characteristic matrix connected with normal and tangent
displacements and the stresses on the boundaries of the layer. In contrast to [1], where the matrices are
represented by series, the results of the present work are expressed in finite form.

Derivation of expressions for characteristics matrices of weakly curved elastic layers is based on using
analogous matrices in the case of plane-parallel, cylindrical, and spherical layers. The latter matrices can
be represented either in the form of series [2, 3] or as finite matrix expressions with elements which contain
hyperbolic and cylindrical functions. The first of these representations was used in [1]. In the present paper
the second expressions are used for the characteristic matrices in three cases: 1) waves along a generator of
a cylindrical layer; 2) azimuthal waves in a cylindrical layer; 3) meridional waves in a spherical layer.

1. Characteristic Matrix of a Weakly Curved Cylindrical
Layer in the Case of Waves along a Generator

Suppose that in a cylindrical coordinate system $r, \theta, z$ there is given a homogeneous, isotropic, elastic
layer $r_1 < r < r_2$ characterized by the density $\rho$ and the Lamé coefficients $\lambda$ and $\mu$. It is assumed that the dis-
placement vector in this layer does not depend on the azimuthal coordinate $\theta$ and does not contain a component
along this coordinate. The field of displacements and stresses in such a layer is represented by the relations

$$
\mathbf{u}_z = \int_0^{\infty} e^{-ikz} \frac{dk}{2\pi i} \int_{\ell_1}^{\ell_2} \mathbf{u}_z(\kappa, \ell, z) e^{ikb} d\ell, \quad \mathbf{u}_\ell = \int_0^{\infty} e^{-ikz} \frac{dk}{2\pi i} \int_{\ell_1}^{\ell_2} \mathbf{u}_\ell(\kappa, \ell, z) e^{ikb} d\ell,
$$

(1.1)

The connection between the values of the matrix

$$
\mathbf{W} = (\mathbf{u}_z, \mathbf{u}_\ell, \mathbf{T}_{zz}, \mathbf{T}_{\ell z})^T
$$

at $r = r_1$ and $r = r_2$ is given by the formula

$$
\mathbf{W}(r_2) = C \mathbf{W}(r_1),
$$

(1.3)

where $C$ is the characteristic matrix in question, and the symbol $T$ denotes the transpose.

On the basis of the Lamé equations and Hooke's law, the characteristic equation can be represented by

$$
C = F(\tau_2)K \overline{F}^{-1}(\tau_1),
$$

(1.4)

in which

$$
F(\tau_2) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad \overline{F}^{-1}(\tau_1) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
$$

(1.5)

and the elements of the matrix $K$ have the expressions

$$
k_{11} = \chi \mu (2d_x H_{11}^x - d_y H_{11}^y), \quad k_{12} = -i\chi \mu (d_x H_{11}^x - 2d_y H_{01}^y), \\
k_{13} = i\chi \mu (H_{00}^x - d_y H_{11}^y), \quad k_{14} = -\chi (d_y H_{01}^x - d_y H_{01}^y), \\
k_{21} = i\chi \mu (2d_x H_{11}^x - d_y H_{11}^y), \quad k_{22} = \chi \mu (2d_y H_{11}^x - d_y H_{11}^y),
$$

(1.6)

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