INTERFERENCE TRANSVERSE SHEW WAVES IN A TRANSVERSALLY, ISOTROPIC, INHOMOGENEOUS HALF SPACE

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In the paper Love waves in a transversally, isotropic, inhomogeneous half space are investigated in the case where the plane of isotropy does not coincide with the boundary plane. The dependence on the angle between them is established in the solution obtained.

We consider the propagation of interference transverse SH surface waves in a transversally, isotropic, inhomogeneous half space in the case where the plane of isotropy of the medium does not coincide with the boundary plane. Suppose that in the coordinates (x, y, z) the half space occupied by the medium is determined by the inequality \( z \rightarrow 0 \). The existence in the plane case of the waves indicated not connected with oscillations of another polarization is possible under the condition that the propagation speeds and the density of the medium do not depend on the coordinate y, while the plane of propagation of the waves passes through the normals to the boundary of the medium and to the plane of isotropy. The equation of motion in this case has the form

\[
a \frac{\partial^2 u}{\partial x^2} + \beta \frac{\partial^2 u}{\partial z \partial x} + c \frac{\partial^2 u}{\partial x^2} + \left( \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} \right) \frac{\partial u}{\partial x} + \left( \frac{\partial b}{\partial x} + \frac{\partial c}{\partial y} \right) \frac{\partial u}{\partial y} + \omega^2 \rho u = 0. \tag{1}
\]

Here we have set

\[
\begin{align*}
a &= \mu \cos^2 \alpha + \mu'' \sin^2 \alpha, \\
b &= (\mu'' - \mu) \sin 2 \alpha \cos \alpha, \\
c &= \mu \sin^2 \alpha + \mu' \cos^2 \alpha,
\end{align*}
\]

\( \mu \) and \( \mu'' \) are the elastic constants, \( \rho \) the density, and \( \alpha \) is the angle of inclination of the plane of isotropy to the boundary plane.

Suppose that the boundary of the half space is free of stresses; then on it the following condition is satisfied:

\[
\tau_{yx} \bigg|_{z=0} = b \frac{\partial u}{\partial x} + c \frac{\partial u}{\partial z} \bigg|_{z=0} = 0. \tag{3}
\]

We shall henceforth consider only proper oscillations of the medium that are concentrated in a neighborhood of the boundary \( z = 0 \). To such oscillations there obviously correspond rays which "do not deviate strongly" from the boundary. It is known that in an anisotropic medium the rays and wave fronts are not orthogonal to one another. This circumstance causes certain special features of the process in question and necessitates seeking a solution not in the traditional form (see, e.g., [1]) but rather in the somewhat different form

\[
u = \sum_{n=0}^{N-1} \omega_n \sum_{j=0}^{N} \left[ \frac{\omega_0(\omega + \omega_j x)z + \omega_j z}{1 + 0(\omega_0)^N} \right] e^{i(\omega_0 x + x z) t},
\]

i.e., the term \( i\omega q(x)z \) is added to the phase of the wave.

Introducing the new coordinate

\[ \zeta = \omega \cdot z \]  

and proceeding according to the usual scheme of the method of [1], to determine the unknown functions \( U_n, p, q, \) and \( r \) contained in expression (4), we obtain the recurrent system of equations

\[ \sum_{n=1}^{\infty} U_{n-1} U_{m+n} = 0, \]

and the boundary conditions

\[ \sum_{n=0}^{m} M_{n} U_{m-n} \bigg|_{\zeta=0} = 0, \]

The operators \( \mathcal{L}_n \) and \( M_n \) contained in them are defined by the equalities:

\[
\begin{align*}
\mathcal{L}_0 U &= (p_0 - a_0 \rho^2 - 2b_0 \rho' q - c_0 q^2) U, \\
\mathcal{L}_1 U &= 2i(b_0 \rho' + c_0 q) \frac{\partial U}{\partial \zeta}, \\
\mathcal{L}_2 U &= c_0 \frac{\partial^2 U}{\partial \zeta^2} - \left[ 2(a_0 \rho' + b_0 q)(\zeta' + q \zeta) + (a_0 \rho'^2 + 2b_0 \rho' q + c_0 q^2 - \rho_0) \zeta \right] U, \\
\mathcal{L}_3 U &= 2i(a_0 \rho' + b_0 q) \frac{\partial U}{\partial \zeta} + 2i \left[ b_0 \zeta' + (b_0 q' + c_1 q + b_1 \rho') \zeta \right] \frac{\partial U}{\partial \zeta} + i(a_0 \rho'^2 + 2b_0 \rho' q + a_1 \rho' + b_0 q + b_1 \rho' + c_1 q) U,
\end{align*}
\]

while the quantities \( a_n, b_n, c_n, \) and \( \rho_n \) are determined by the expansions

\[
\begin{align*}
a &= a + \omega^{-\frac{i\pi}{2}} a_1 \zeta + \cdots, \\
b &= b + \omega^{-2\pi} b_1 \zeta + \cdots, \\
c &= c + \omega^{-3\pi} c_1 \zeta + \cdots, \\
\rho &= \rho_0 + \omega^{-4\pi} \rho_1 \zeta + \cdots.
\end{align*}
\]

The first two equations of system (6) are satisfied if the following equalities hold:

\[
\begin{align*}
a_0 \rho'^2 + 2b_0 \rho' q + c_0 q^2 &= \rho_0, \\
b_0 \rho' + c_0 q &= 0.
\end{align*}
\]

We introduce the function

\[ \Phi(x, z) = \sqrt{\frac{\rho c}{\alpha \omega - b^2}} = \sqrt{\frac{\rho c}{\mu \omega}} = \sqrt{\frac{\rho c}{\mu x}} \sin^2 \omega + \frac{\rho}{c} \cos^2 \omega \].