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LITERATURE CITED


ESTIMATE OF PERTURBATIONS IN SOLVING NONUNIFORM NEUTRON TRANSPORT PROBLEMS BY THE MONTE CARLO METHOD

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As computers develop there is a steadily increasing interest in using the Monte Carlo method to model radiative transport processes. The present paper gives computational formulas for an important class of reactor physics problems, the calculation of nonlocal perturbations of linear functionals. We consider a group representation of the dependence of neutron cross sections on energy. We describe the case of finite perturbations resulting from a limiting transition when the perturbation parameter tends to zero, and we obtain formulas for the derivatives of linear functionals (the sensitivity coefficients). The dispersion of the computational scheme is studied in an example of a homogeneous one-velocity medium.

The paper presents typical accuracies in calculating the Doppler coefficients of reactivity and density reactivity effects by the Monte Carlo method in reactors and cores of various types.

Method of Correlated Trajectories. We assume that all the medium cross sections are functions of a certain parameter \( \alpha \). With \( \alpha = 0 \) we obtain a medium in which the particle motion is random. The "effective weight" of a particle belonging to a medium with \( \alpha \neq 0 \) but following a trajectory drawn by lot in a medium with \( \alpha = 0 \), we shall designate \( W \). By \( X_n \) we denote the set of particle phase coordinates \( (r, E, \Omega) \) at the \( n \)-th collision. The energy \( E \) and the flight angle \( \Omega \) are taken at the moment of collision. The effective particle weight \( W_n \) at the \( n \)-th collision is determined as follows:

\[
W_n = \frac{S(X_n, \alpha)}{S(X_n, 0)} \prod_{i=1}^{n-1} \exp (-\Delta \tau_i) \frac{\Sigma_i(X_i, \alpha) w_i(E_i, \Omega_i - E_{i+1}, \Omega_{i+1}, r_i, \Omega_i)}{\Sigma_i(X_i, 0) w_i(E_i, \Omega_i - E_{i+1}, \Omega_{i+1}, r_i, 0)} \exp (-\Delta \tau_i) \frac{\Sigma_i(X_n, \alpha)}{\Sigma_i(X_n, 0)}
\]

i.e., from the definition \( \prod_{i=1}^{n} f_i = 1 \). In Eq. (1) \( \frac{S(X_n, \alpha)}{S(X_n, 0)} \) is the source ratio; \( \Delta \tau_i = \int_{t_{i-1}}^{t_i} [\Sigma (r, \alpha) - \Sigma (r, 0)] \, dt \) is the perturbation of optical depth between the points of collisions \( i-1 \) and \( i \); \( \Sigma(X, \alpha) \) is the total cross section.

for the medium if elastic scattering of the particle is chosen isotropically in the laboratory coordinate system, and the transport cross section is taken as \( \Sigma(X, \alpha) \); \( \Sigma_s(X, \alpha) \) is the scattering cross section*; and \( \Sigma_g(E_1, Q_1 \rightarrow E_{i+1}, Q_{i+1}, r_i, \alpha) \) is the angular energy index corresponding to the type of scattering. The values of \( \Sigma_s \) and \( \Sigma_g \) in the numerator and denominator of Eq. (1) are chosen for the specific scattering process occurring in the medium, where the random process holds (a medium with \( \alpha = 0 \)). The ratio \( \frac{\Sigma_g(x)}{\Sigma_s(y)} \) must be finite (a medium with \( \alpha = 0 \) must not be "narrower" than a medium with \( \alpha \neq 0 \)) in a multitude of possible values of the particle phase coordinates \( X \).

We now turn to a detailed determination of the ratio \( \nu = \frac{\Sigma_g(x)}{\Sigma_s(y)} \) in a group model for the dependence of cross section on energy \([2]\): \( \nu = \frac{\Sigma_g(x)}{\Sigma_s(y)} \) if the neutron remains in the same group \( \Gamma \), where \( \Sigma_g(\alpha) \rightarrow \Gamma \) is the reaction cross section for "remains in the same group"; \( \nu = \frac{\Sigma_g(x)}{\Sigma_s(y)} \), if the neutron transfers to the next group \( \Gamma + 1 \), where \( \Sigma_s(x) \) is the cross section for transition to the next group.

There is a continuous degradation of neutron energy: 1. The reactions are selected from the macroscopic medium constants. If the neutron transfers to the next energy group after being scattered, then \( \nu = \frac{\Sigma_s(x)}{\Sigma_s(y)} \), where \( \xi \) is the average loss of lethargy; and \( \Sigma_s(\alpha) \) is the elastic scattering cross section. If the neutron remains in the same group, then \( \nu = \frac{\Sigma_s(x)}{\Sigma_s(y)} \), where \( \Delta \) is the group lethargy interval.

2. We determine the nuclide \( i \) in which the scattering occurs. \( \nu = \frac{\Sigma_s(x)}{\Sigma_s(y)} \); and \( \Sigma_s(\alpha) \) is the scattering cross section for the \( i \)-th nuclide.

For inelastic scattering and the \( n-2n \) reaction \( \nu = \frac{\Sigma_s(x)}{\Sigma_s(y)} \); and \( \Sigma_s(n-2n) \) is the cross section for an inelastic transition (or the \( n-2n \) reaction) from group \( i \) to group \( j \).

If there is an energy degradation on a continuous scale in the scheme of random motion, then \( \nu = \frac{\Sigma_s(x)}{\Sigma_s(y)} \) for hydrogen scattering (with cross section \( \Sigma_{sH} \)). In the group approach \( \nu \) is determined in the same way as for inelastic transitions, as a ratio of the corresponding matrix elements \([3]\). However, in this case, since the matrix elements are not perturbed, \( \nu = \frac{\Sigma_s(x)}{\Sigma_s(y)} \).

For fission events \( \nu = \frac{\Sigma_s(x)}{\Sigma_s(y)} \), where \( \Sigma(X, \alpha) \) is the spectrum of ejected particles.

The perturbation of the linear flux functional

\[
\Delta I = \int \left[ \Sigma_p(\alpha) \Phi(x) - \Sigma_p(0) \Phi(0) \right] dx,
\]

obtained by evaluating the collisions, can be calculated from the formula \([1]\)

\[
\Delta I = \frac{1}{H} \sum_{i=1}^{H} \sum_{n=1}^{I_i} \left[ \Sigma_p(X_n, \alpha) - \Sigma_p(X_n, 0) \right],
\]

where \( \Sigma_p(X_n, \alpha) \) is a certain weight function, e.g., \( \nu \Sigma_f \), if the multiplication coefficient is determined \([3]\); \( I_i \) is the number of collisions in the \( i \)-th history; \( \Phi \) is the flux; and \( H \) is the number of histories considered.

The evaluation of Eq. (2) is unbiased, since the minuend and the subtrahend are not biased \([4]\).

Evaluation of Derivatives of the Linear Functionals. We can obtain a formula for the derivatives \( \frac{\partial I}{\partial \alpha} \) directly, by going to the limit \( \lim_{\Delta \alpha \to 0} \frac{\Delta I}{\Delta \alpha} \) in Eq. (2) \([5,6]\). The analog of the effective weight \( W \) is the "differential weight" \( D \), obtained by differentiating Eq. (1) with respect to \( \alpha \):

\[
D_n = \frac{\Sigma_s(X_n)}{\Sigma_s(Y_n)} + \sum_{i=1}^{n-1} \left[ \Delta \tau_i \frac{\Sigma_s(X_n)}{\Sigma_s(Y_n)} + \frac{\nu_i(E_i, Q_i \rightarrow E_{i+1}, Q_{i+1}, r_i)}{\nu_i(E_i, Q_i \rightarrow E_{i+1}, Q_{i+1}, r_i)} \right] \Delta \tau_i + \frac{\Sigma_s(X_n)}{\Sigma_s(Y_n)}.
\]

The primes denote derivatives with respect to \( \alpha \) (for \( \alpha = 0 \)).

* The following types of scattering (the number of particles need not be conserved) we usually selected in reactor calculations by the Monte Carlo method: elastic, inelastic, \( n-2n \) reactions, hydrogen scattering, and fission of fuel nuclei with ejection of secondary neutrons.