The reliable estimation of indirect-measurement errors requires knowledge of the cross-correlation moments of the errors of the particular measuring devices [1]. The set of all such moments comprises the values of the error cross-correlation function (ECCF) for the measuring instruments, which needs to be known and must therefore be monitored.

We discuss here a procedure for obtaining estimates of the instrument ECCF when the information-yielding parameter of the instrument output signal is a time interval. We interpret a measuring device (or, simply, instrument) in the ensuing discussion to be such an instrument. For brevity we say instead of the expression "information-yielding parameter of the output signal," simply, "output signal."

The proposed procedure is equally valid for the instrument error autocorrelation function, which can be treated formally as a special case of the ECCF when both instruments are identical and the nominal values of their output signals are the same. Examples of instruments are time-marker generators, time interval-to-time interval converters, and voltage-to-time interval converters. An example of a converter of the first type is a magnetograph that plots pulse-frequency-modulated and pulsewidth-modulated signals, and an example of the second type is a pulsewidth modulator.

The instrument ECCF $R_{\Delta_1,\Delta_2}^\circ (\lambda_{1,2})$ is defined as the function

$$R_{\Delta_1,\Delta_2}^\circ (\lambda_{1,2}) = \Omega_1(t) \Omega_2(t+\lambda_{1,2}) = F(\bar{T}_1, \bar{T}_2, \bar{\lambda}_{1,2}).$$

(1)

where $T_1$, $T_2$, and $\lambda_{1,2}$ are the instrument output signals and delay times (Fig. 1) and $\hat{\Delta}_1$ and $\hat{\Delta}_2$ are the centered instrument errors at the points $T_1$ and $T_2$ of their respective measurement ranges. These errors can be for two instruments, for two channels I and II of a multichannel instrument, or errors of a single-channel instrument.

The instrument ECCF is conveniently represented as a function of the input signals, because the instrument monitoring process yields samples of random error values for a given input signal, where a succession of "identical" input signals is delivered to the instrument input and random output signals are measured. It is virtually impossible to obtain samples of random error values for a fixed output signal.

The ECCF in (1) is written as a function of the mathematical expectations $\bar{T}_1$ and $\bar{T}_2$ of the output signals, and this is consistent with its determination as a function of the input signals. In the investigated instruments the delay is a variable of the same kind as the output signal, i.e., a time interval. The presence of random instrument error causes the delay between output signals to be a random variable for given input signals and delay between them. On the same grounds as stated above for the output signals the argument of the ECCF (1) is the expectation value of the delay $\bar{\lambda}_{1,2}$.

The instrument error autocorrelation function is a special case of the ECCF wherein $T_1$ and $T_2$ are measured by the same instrument (or same instrument channel) and $\bar{T}_1 = \bar{T}_2$.

Fig. 1. MI = measuring instrument.
A standard procedure for monitoring the ECCF is to determine a certain set of its values, i.e., estimates of the cross-correlation moments, and to plot the function according to those estimates [2]. However, if one is concerned with monitoring the ECCF or instruments of the type in question and monitoring the ECCF over a wide range of values of $\tau_1$, $\tau_2$, and $\lambda_{12}$, this procedure becomes impractical. The reason lies in the enormous difficulty of such a monitoring operation; the instrument ECCF is a function of three variables: $\tau_1$, $\tau_2$, and $\lambda_{12}$, and in order to estimate it over a wide range of values of $\tau_1$, $\tau_2$, and $\lambda_{12}$, it is necessary to find a large number of error cross-correlation moments, and to determine just one such moment is a difficult enough operation.

The following, simpler procedure can be used to determine the instrument ECCF. The beginning and end of the time interval $T$ is usually marked according to the instant at which the leading edge of a voltage pulse $U$ crosses a threshold level $U_t$ (Fig. 2a). For two instrument channels (or two instruments) it is possible to specify and measure the time interval $T_{mu}$ between the leading edges of the pulses in channels I and II (Fig. 2b). We define the mutual error of the instrument(s) as the difference between the nominal value $T_{mu,n}$ and the actual value $T_{mu}$ of this mutual time interval:

$$\Delta_{mu} = T_{mu,n} - T_{mu}$$  \hspace{1cm} (2)

Now the expression for the instrument ECCF can be written in the form

$$R_{\Delta_1, \Delta_2} [\lambda_{1,2}] = \Delta_1(t) \Delta_2(t + \lambda_{1,2}) = (1/2) [(\Delta_2 + \Delta_{1,2})^2 +$$

$$+ (\Delta_{1,2} - \Delta_1)^2 - (\Delta_{1,2})^2 - (\Delta_2 + \Delta_{1,2} - \Delta_1)^2]$$,  \hspace{1cm} (3)

where

$$\Delta_{1,2} = \lambda_{1,2} - \lambda_{1,2} = (\lambda_{1,2, n} - \lambda_{1,2}) - (\lambda_{1,2, n} - \lambda_{1,2})$$  \hspace{1cm} (4)

Relation (3) is proved with each term on its right-hand side represented by the sums of the mean squares and twice the product means:

$$(\Delta_2 + \Delta_{1,2})^2 = (\Delta_2)^2 + (\Delta_{1,2})^2 + 2\Delta_2 \Delta_{1,2} \text{ etc.}$$,  \hspace{1cm} (5)

followed by algebraic summation according to (3).

According to (3) the ECCF $R_{\Delta_1, \Delta_2} [\lambda_{1,2}]$ is equal to half the algebraic sum of four terms, which represent values of a single quantity, namely the variance of the instrument mutual error. Thus, the delay $\lambda_{1,2}$ is a time interval of the type $T_{mu}$ whose beginning is marked by a signal in one channel (voltage drop), and its end by a signal in the other channel (see Figs. 1 and 2).

By the definition of the mutual error (2) and (1) the quantity $\Delta_{1,2}$ (4) represents the centered mutual error of the instrument subject to the delay expectation value $\lambda_{1,2}$, so that the third term in the bracketed expression in (3) is the variance of the instrument mutual error for $T_{mu} = \lambda_{1,2}$; the first term is the variance of the mutual error for $T_{mu} = T_1 + \lambda_{1,2}$, etc.

If we denote by $\varphi(T_{mu})$ the dependence of the variance of the instrument mutual error on $T_{mu}$, we can then represent (3) in the form

$$R_{\Delta_1, \Delta_2} [\lambda_{1,2}] = (1/2) [\varphi(T_2 + \lambda_{1,2}) + \varphi(\lambda_{1,2} - T_2)] - \varphi(\lambda_{1,2}) - \varphi(|T_2 + \lambda_{1,2} - T_2|)$$.  \hspace{1cm} (6)