EFFECT OF COUNTERFLOW MIXED CONVECTION ON
THE HEAT TRANSFER IN VERTICAL LIQUID-METAL
HEAT EXCHANGERS

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The results are shown here of an experimental and analytical study concerning the heat transfer between sodium and sodium-potassium alloy in counterflow heat exchangers with counterflow mixed convection in one of the ducts. It is shown that, at low Pe numbers, the heat transfer is determined by the effect of longitudinal heat overruns in the liquid.

Counterflow mixed convection of heat is observed in a liquid heated while moving downward or cooled while moving upward through ducts of large cross sections. This mode of heat transfer is characterized by the presence of a large turbulence component $\lambda_{TX}$ in the longitudinal thermal conductivity of the liquid — a result of the positive vertical gradient of liquid density and, as a consequence, of large heat overruns along the height (according to indirect estimates based on the data in [2], $\lambda_{TX}/\lambda$ may equal to 10 or more). This makes the true "liquid-to-liquid" temperature drop $\Delta t$ in the heat exchanger much smaller than $\Delta t_p$ calculated outside the heat transfer zone. The effect of longitudinal heat overruns in a heat exchanger has been studied in [1, 2] for the case $\lambda_{TX} = 0$. The equations describing the change in the mean temperature of the liquid in the ducts at $W_1 = W_2$ are then:

$$t'_1 - \frac{Pe}{\kappa} t'_1 + \frac{4\nu_h}{\kappa} (t_2 - t_1) = 0, \quad (1)$$

$$t'_2 + Pe t'_2 - 4\nu_h (t_2 - t_1) = 0. \quad (2)$$

The boundary conditions are

$$X = 0 \; \begin{array}{ll} a) \; t_1 = \frac{\kappa}{Pe} t'_1, & b) \; t'_2 = 0; \end{array} \quad (3)$$

$$X = L \; \begin{array}{ll} c) \; t'_1 = 0, & d) \; t_2 = t_0 - \frac{t'_2}{Pe}. \end{array}$$

Equations (1) and (2) apply where the wall across which heat transfer occurs has a negligibly small longitudinal component of thermal conductivity and the geometrical dimensions of the ducts are constant over their active lengths.

When $\lambda = \text{const} \neq 0$ in one of the heat exchanger ducts (e.g., in the one denoted by subscript 2), then the respective equations become

$$t'_1 - \frac{Pe}{\kappa + \Lambda} t'_1 + \frac{4\nu_h}{\kappa + \Lambda} (t_2 - t_1) = 0, \quad (4)$$

$$t'_2 + \frac{Pe}{1 + \Lambda} t'_2 - \frac{4\nu_h}{1 + \Lambda} (t_2 - t_1) = 0. \quad (5)$$

Obviously, these equations will be identical to Eqs. (1), (2) if in the latter ones $Pe$, $\nu_h$, and $\kappa$ are replaced by $Pe/(1 + \Lambda)$, $\nu_h/(1 + \Lambda)$, and $\kappa/(1 + \Lambda)$ respectively.
We will consider the case where \( x/(1 + \Lambda) \approx 0 \) and Eq. (4) is

\[
  t_1 - \frac{4Nuk}{Pe} (t_2 - t_1) = 0,
\]

while boundary condition (3c) becomes meaningless. Solving (5) and (6) simultaneously, we obtain

\[
  t_1 = \frac{t_o}{H_o} \left[ 1 - \exp \beta X + \left( 1 + \frac{Pe}{4Nuk} \beta \right) \beta X \right],
\]

\[
  t_2 = \frac{t_o}{H_o} \left[ 1 + \left( 1 + \frac{Pe}{4Nuk} \beta \right) \exp \beta X + \left( 1 + \frac{Pe}{4Nuk} \beta \right) \left( \frac{Pe}{4Nuk} + X \beta \right) \right],
\]

\[
  H_o = \beta \left[ 1 + \beta \frac{Pe}{4Nuk} \right] \left[ \frac{Pe}{4Nuk} + \frac{1 + \Lambda}{Pe} + L - \left( \frac{1}{\beta} + \frac{1 + \Lambda}{Pe} \right) \exp \beta L \right].
\]

Let \( \pi = 4Nuk/Pe^2 (1 + \Lambda) \). Then

\[
  \Delta t = \frac{t_o}{H_o} \left( 1 + \frac{1}{\pi} \right) \left( \frac{1}{\pi} + \exp \beta X \right),
\]

\[
  \Delta t_p = \frac{t_o}{H_o} \left( \frac{1 + \pi}{\pi} \right)^2.
\]

There are two ways of calculating the quantity of heat \( Q \) transferred in the heat exchanger: \( Q = K \Delta t \) and \( Q = K_p \Delta t_p \). In the second case one tentatively includes the effect of longitudinal heat overruns in the heat transfer coefficient while the temperature drop is assumed to remain without change. Then

\[
  Nu_{cr} = \frac{K_p d_s}{\lambda_s} = \frac{\Delta t}{\Delta t_p} \quad Nu_h = \frac{Nuk}{1 + \pi} + \frac{\pi^2}{(1 + \pi)^2} \cdot \frac{(1 - \exp \beta L)}{4L} Pe.
\]