Shutdown conditions for a reactor are discussed ("optimum control") which, by adjusting the reactor power over a specified interval of time (of the order of several hours), enables the maximum value of the reactor poisoning to be minimized.

The problem of finding the optimum control has been solved approximately by a method developed by R. Fedorenko for the numerical solution of nonlinear problems of optimum control. This method is an iteration procedure in which a certain initial (not optimum) control is successively varied with the aim of reducing the value of the functional. The procedure leads quite rapidly to a control which is sufficiently close to optimum. Analysis of a large number of numerical solutions enables a very plausible assumption to be made concerning the simple structure of the precise solution of the problem.

For a reactor with a high thermal neutron flux, an excess of reactivity which is adequate to compensate for iodine poisoning can be achieved in principle by increasing the reactor charge. However, this leads to a complex control system and gives rise to a number of difficulties associated with ensuring the operating safety of the reactor and, moreover, it has a negative effect on the efficiency characteristics of the assembly. This approach to the problem of iodine poisoning is possible only in the case of relatively low neutron fluxes, since compensation for the xenon which appears after reactor shutdown at high fluxes may require a severalfold increase of the reactor loading compared with the critical loading. The principle of "adding" to the active zone after reactor shutdown has essentially the same disadvantages, apart from leading to considerable complexity of construction.

However, the xenon concentration in a reactor is extremely sensitive to change of neutron flux. This gives a basis for hoping that by means of a carefully controlled change of reactor power prior to shutdown, a significant reduction in xenon concentration can be achieved. This type of problem was posed for the first time in 1959. In [1] the possibility of using dynamic programming methods in order to solve the problem of finding the optimum shutdown conditions for a reactor was considered. However, the method developed in this project required such a large operative memory of the electronic computer used for solving the problem that obviously, at that time, it could not be used (no report on the completion of the project has so far appeared).

In [2], the results of calculations on an analog computer are described for the change of xenon concentration with specified laws of reactor power reduction. It was shown that for all the laws selected for varying the power (linear, exponential, and with constantly increasing slope of the curve), some reduction could be achieved in the maximum xenon concentration with a simultaneous increase in the time during which the xenon concentration increased to the equilibrium value. Since, in this report, attempts to find and investigate the optimum conditions for reactor shutdown were not undertaken, the question concerning the possibilities of this method must remain open.

In the present paper, an account is given of the solution of the problem of finding the optimum reactor shutdown conditions by means of an approximate method worked out by R. Fedorenko for solving numerically nonlinear problems of optimum control. This method is essentially a direct method of solving variational problems, the use of which enables one to find quite rapidly the control sufficiently close to optimum (20-30 min of arithmetical operations), without imposing large demands on the operative memory of the computer. The analysis of a large number of numerical solutions has enabled a very plausible assumption to be made concerning the simple structure of the precise solution to the problem.
Statement of the Problem

In describing the change of xenon concentration, a simplified model was used, based on the assumption that the neutron flux averaged with respect to the fuel volume determines the fission product concentration. On this assumption, and using universal symbols [3], the equations for the change of iodine and xenon concentrations can be written as:

\[
\begin{align*}
\frac{dY}{dt} &= \gamma_1 \Sigma_f \Phi + \lambda_1 Y, \\
\frac{dX}{dt} &= \gamma_2 \Sigma_f \Phi + \lambda_1 Y - (\lambda_2 + \sigma_2 \Phi) X.
\end{align*}
\]

(1)

The following limitations \( \Phi \) are superimposed on the thermal neutron flux:

\[
\begin{align*}
\Phi (t) &= \Phi_0, & t &\leq 0, \\
0 &\leq \Phi (t) \leq \Phi_0, & 0 &< t < T, \\
\Phi (t) &= 0, & T &\leq t,
\end{align*}
\]

(2)

where \( T \) is a specified control time.

The time interval from 0 to \( T\Phi(t) \) should be chosen such that the functional

\[
F(\Phi) = \max_{0 \leq t < \infty} X(t).
\]

(3)

is minimized. It will be explained subsequently that it is more convenient to take not \( \phi(t) \), but

\[
\tilde{\Phi} (t) = \frac{d\Phi}{dt},
\]

(4)

as the unknown equation, so that the minimized functional is

\[
F(\tilde{\Phi}) = \max_{0 \leq t < \infty} X(t).
\]

(3')

Although the statement of the problem is more complex than in [1], it proved to be more convenient to apply the numerical method for solution of the problem.

A limitation on the rate of change of flux with time must be added to the equations which describe the problem. However, this limitation has no significant effect on the solution of the problem by the method described, since the time interval used is, as a rule, considerably less than the attainable asymptotic periods of the reactor. Moreover, this limitation would lead to the appearance of an additional parameter and would complicate the analysis and the generality of the results obtained. Consequently, this limitation was not imposed. The conditions obtained should be considered as ideal and should aim toward realization, taking into account the special features of the kinetics of each reactor.

The solution of the problem as stated depends formally on the four parameters \( E_1, \sigma_2, \Phi_0, \) and \( T \). The decay constants \( \lambda_1 \) and \( \lambda_2 \) are generally unchanged and the yield of iodine and xenon (\( \gamma_1 \) and \( \gamma_2 \), respectively), vary only as a result of transition from one type of fissionable nucleus to another. It is not difficult to see that by transforming to dimensionless quantities

\[
y = \frac{Y}{Y_0}, \quad x = \frac{X}{X_0}, \quad \varphi = \frac{\Phi}{\Phi_0},
\]

(5)

where

\[
Y_0 = \frac{\gamma_1 \Sigma_f \Phi_0}{\lambda_1}, \quad X_0 = \frac{(\gamma_1 + \gamma_2) \Sigma_f \Phi_0}{\lambda_2 + \sigma_2 \Phi_0}
\]

(6)

are the solutions of system (1) for \( t \leq 0 \), the solution of the problem will depend only on the two parameters \( T \) and \( Z = \sigma_2 \Phi_0 / \lambda_2 \).

Thus, taking into account the symbols used, the solution of the system of equations will have the form