DETERMINISTIC AND STOCHASTIC APPROXIMATING PROBLEMS WITH APPLICATIONS TO PRODUCTION CONTROL

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Improper mathematical programming problems are analyzed and deterministic and stochastic approaches to correcting these problems are suggested. Numerical experiments with test examples are presented. The paper focuses on numerical analysis of improper linear programming problems [1-4], which arise in the context of scarce resources in economics [5]. Parametrization is applied to examine one of the possible approaches to approximation of improper LP problems under deterministic and stochastic conditions. Although the main focus is on improper problems of the 1st kind, we also touch upon some issues connected with improper problems of 2nd and 3rd kind [1]. The analysis of improper LP problems is based on duality theory [2]. Some results specialize the ideas previously presented in [1, 6]. The present paper is a continuation of [4].

INTRODUCTION

The application of mathematical-economic models for solving production control problems has shown that contradictory models are a fairly common occurrence. One of the manifestations of a contradictory model is inconsistency of the constraint system. The reasons for inconsistency are to be sought in scarcity of resources, violations of raw material delivery deadlines, fuzziness and unreliability of economic information, contradicting requirements imposed on the model, and many other factors [1].

The existing software packages (e.g., LP ASU and SPO MPR-2) do not include options for correcting a contradictory model. Moreover, these packages have been designed for RYaD-2 ES computers, which are currently being displaced by IBM PC AT and XT compatible personal computers. Personal computers are becoming very popular and are penetrating into various spheres of human activity.

The transition to market economy focuses the attention on the development of mathematical models that are not only capable of "resolving" the contradictions but also can function under conditions of risk [7]. Risk is the typical case for most economic production systems that have to meet state procurement orders while simultaneously satisfying the demands of free entrepreneurship. In other words, mathematical-economic models are needed that can respond flexibly to uncertainty and varying conditions.

Models of this kind arise in production control. They extend the theory of contradictory LP models toward better and more complete utilization of inexact input information [8, 9]. The results obtained for improper LP problems provide a new view of production control systems and some additional information about economic production systems.

DETERMINISTIC APPROXIMATING PROBLEMS

Consider the problem of finding

\[ x^* = \arg \max \{c^T x : Ax \leq b, x \geq 0 \}, \]

where \( c \in \mathbb{R}^n, b \in \mathbb{R}^m, A = (a_{ij})_{m \times n}, a_{ij} \in \mathbb{R}^1, x \in \mathbb{R}^n, X := \{x: Ax \leq b, x \geq 0\} \) is a convex body in \( \mathbb{R}^n \). The parameters of problem (1) have a traditional economic interpretation in the production context: \( c \) is the product price vector, \( A \) is the (full rank) technology matrix, \( b \) is the resource vector, and \( x \) is the production level vector.

Assume that (1) is an improper LP of the 1st kind [1]. This problem can be parametrized in a certain way. Consider the parametrization in the form \( x^* = \arg \max \{c^T x : Ax \leq b + \Delta b^+ - \Delta b^-, x \geq 0\} \), where the parameters are \( \Delta b^+ \) and \( \Delta b^- \), i.e., the resource expansion vector and the resource contraction vector, respectively \( (\Delta b^+, \Delta b^- \in \mathbb{R}^m_+) \). As the performance criterion (the approximation accuracy criterion) we take the function

\[
\phi(\Delta b^+, \Delta b^-) = (r^+)^T \Delta b^+ - (r^-)^T \Delta b^-,
\]

where \( r^+ \) is the vector of costs associated with acquiring an additional unit of the resource \( (r \in \mathbb{R}^m_+) \), \( r^- \) is the vector of income from saving one unit of the resource \( (r^- \in \mathbb{R}^m_+) \). Then the approximating problem is written in the form

\[
\tilde{x}^* = \arg \min \{(r^+)^T \Delta b^+ - (r^-)^T \Delta b^- : Ax \leq b \Delta b^+ - \Delta b^-, x, \Delta b^+, \Delta b^- \geq 0\},
\]

where \( \tilde{x}^T := (x^T | (\Delta b^+)^T | (\Delta b^-)^T) \) is a block-partitioned vector.

If \( \Delta b^{+,+} \Delta b^{-,-} \subseteq \tilde{x}^* \), then the corrected problem for (1) is the problem of finding

\[
x^* = \arg \max \{c^T x : Ax \leq b + \Delta b^+ - \Delta b^-, x \geq 0\}.
\]

Correction of the problem (1) by the criterion (2) thus reduces to solving the problems (3) and then (4). In this case, the improper LP problem is analyzed in two stages: we first correct the model (by parametrization) and then solve the corrected problem. These two correction stages can be reduced to one stage by combining the problems (3), (4) into a problem of finding

\[
\tilde{x}^* = \arg \max \{c^T x - (r^+)^T \Delta b^+ + (r^-)^T \Delta b^- : Ax \leq b + \Delta b^+ - \Delta b^-, x, \Delta b^+, \Delta b^- \geq 0\}.
\]

Alongside the problem (5), we consider the following varieties:

\[
\tilde{x}^* = \arg \max \{c^T x - (r^+)^T \Delta b^+ + (r^-)^T \Delta b^- : Ax \leq b + \Delta b^+ - \Delta b^-, x \geq 0\};
\]

where \( \Delta b^-, \Delta b^- \) are the lower bound vector on resource contraction and the upper bound vector on resource expansion, respectively \( (\Delta b^-, \Delta b^- \in \mathbb{R}^m_+) \);

\[
\tilde{x}^* = \arg \max \{c^T x - (r^+)^T \Delta b^+ + (r^-)^T \Delta b^- : Ax \leq b + \Delta b^+ - \Delta b^-, x \geq 0\};
\]

The corresponding duals of problems (5)-(8) are

\[
u^* = \arg \min \{b^T u : A^T u \geq c, r^- \leq u \leq r^+\},
\]

where \( u \in \mathbb{R}^m_+ \);

\[
u^* = \arg \min \{b^T u + \Delta \tilde{b}^T v + \Delta \tilde{b}^T w : A^T u \geq c, r^- + v - w \leq u \leq r^+ + v - w, u, v, w \geq 0\},
\]

where \( u^T := (u^T | v^T | w^T) \).