THE $P_3$ APPROXIMATION FOR GASES AND VACUUM

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In calculations on fuel pins for a gas-cooled reactor, it is necessary to consider the neutron distribution in a medium of low density, where $\Sigma t \ll 1$, in which $\Sigma$ is the characteristic dimension and $\Sigma t$ is the total macroscopic cross section for the interaction of neutrons with matter. The equations for the $P_3$ approximation can then be written formally and solved, but considerable errors can arise in numerical implementation from the use of $I_m(x)$, $K_m(x)$ with very small values of the argument. It is therefore desirable to consider the $P_3$ approximation for vacuum, i.e., for $\Sigma t = 0$. However, the $P_N$ approximation causes certain difficulties in obtaining a closed system of equations for the moments of the neutron distribution in vacuum, although this complication vanishes if the pattern has azimuthal symmetry. The system then becomes two independent ones: for the even and odd moments (the number of equations in each coincides with the number of unknowns). It is shown here that the $P_3$ approximation gives good results for cells having cylindrical vacuum zones surrounded by the medium.

Neutron Pattern in a Zone Containing Vacuum

The neutron distribution in the $P_3$ approximation can be written in the following form for cylindrical geometry for an azimuthally symmetrical field (it is also assumed that the field is symmetrical with respect to the angle $\phi$):

$$f(r, \Omega) = \frac{1}{\sqrt{2\pi}} \sum_{l=0}^{3} \sum_{m=-l}^{l} \left(1 - \frac{\delta_{lm}}{2}\right) F_{l,m}(r) \tilde{P}_{lm}(\cos \Psi) \cos m\phi,$$

where $\Omega(\phi, \Psi)$ is unit vector in the neutron motion direction, which is defined by the polar coordinates, $\phi$ is the angle between the radial direction $r$ and the projection of $\Omega$ on the $z=0$ plane $\Omega_{xy}$, $\Psi$ is the angle between the positive direction of the cylindrical $z$ axis and $\Omega$, and $\tilde{P}_{lm}(\cos \Psi)$ are adjoint Legendre functions normalized as in [1]. The system of linear differential equations for the moments $F_{l,m}(r)$ is readily derived for the vacuum case from the corresponding system for a medium [1] by putting $\Sigma t = 0$. The solution is the flux vector $\Psi(r) = \text{col} \{1/2 F_{0,0}(r), 1/2 F_{2,0}(r), F_{2,2}(r)\}$ together with the current vector $j(r) = \text{col} \{F_{1,1}(r), F_{3,1}(r), F_{3,3}(r)\}$, which can be put in matrix form:

$$\Psi(r) = \hat{I}(r) B;$$

$$j(r) = \hat{K}(r) A,$$

where

$$\hat{I}(r) = \begin{pmatrix} 1 & 0 & -\sqrt{\frac{3}{5}} r^2 \\ 0 & 1 & (1/\sqrt{5}) r^2 \\ 0 & 0 & r^2 \end{pmatrix};$$

$$\hat{K}(r) = \begin{pmatrix} 1/r & 0 & 0 \\ 0 & 1/r & (1/\sqrt{15}) r \\ V14/15/r & (-1/\sqrt{15})/r & 1/r^3 \end{pmatrix};$$

in which $A$ and $B$ are constant three-component vectors defined by the boundary conditions. At the boundary with the vacuum surrounded by the medium, we assume that the neutron flux density is continuous, as previously [2]. The solution in the internal region (within the empty cylindrical region) is defined only by (2), while that in the outer one is defined by (3). The first assertion follows from the boundedness of the neutron density at $r = 0$, while the
second is due to it decreasing at infinity. This is so because the number of neutrons in a
cylindrical layer of vacuum of thickness \( v \) (\( v = \text{constant} \) is the neutron velocity) at a large
distance from the cylinder, where its size can be neglected, is independent of the radius of
the layer and is equal to the number of neutrons emitted by the cylinder into vacuum in unit
time. As the volume of the layer increases with its radius (thickness fixed at \( v \)), the mean
neutron density falls as the distance increases. All the above applies for any \( P_N \)
approximation. Only the dimensions of matrices (4) and (5) alter, which are always upper and lower
triangular matrices correspondingly and include the matrices from the lower approximations.
By analogy with [3], we organize a matrix fitting for a multizone fuel-pin assembly. Any zone
containing vacuum may be divided into layers, for which the matrix-fitting formulas also apply.

Neutron Pattern in the Central Zone of a Cluster with Vacuum between Rods

The kinetic equation is linear, so the neutron distribution at any point in the central
zone of a cluster outside the rods can be represented as a superposition of the inherent
fields for all the rods in the central zone and the field induced by the surface of the central
zone itself (i.e., a field within the cylindrical region without rods). To write the
boundary conditions at the outer surface of any rod, we need to transfer the field from the
boundary of the central zone to the coordinate system of this rod. As azimuthal symmetry is
assumed for the fuel-pin and rod pattern, we average it along the boundary of a rod. The
field induced by the boundary of the central zone containing vacuum is determined only by the
flux vector (\( \mathbf{j}(r) = 0 \) for the internal problem), and it takes the following form after aver-
ingar over the boundary of rod \( k \):

\[
\bar{\mathbf{q}}_{kc}(\rho) = \hat{\mathbf{f}}_{kc}(\rho) \mathbf{B},
\]

where

\[
\hat{\mathbf{f}}_{kc}(\rho) = \begin{pmatrix} 1 & 0 & -\sqrt{\frac{\rho}{6}} \left( \rho^2 + Z^2 \right) \\ 0 & 1 & \frac{1}{\sqrt{\rho}} \left( \rho^2 + Z^2 \right) \\ 0 & 0 & \rho^2 \end{pmatrix};
\]

in which \( \rho \) is rod radius and \( Z \) is the distance between the centers of the cluster and rod \( k \).
For the same purpose (writing the boundary conditions at the surface of a rod), we consider
the field from all rods in the coordinate system of rod \( k \) and average along its boundary. As
the field from any rod is represented only by the current vector (\( \mathbf{q}(r) \equiv 0 \) for the external
problem), we get the following: the averaged effect of rod \( k' \) at the boundary of rod \( k \) is 0,
i.e., \( \bar{\mathbf{j}}_{kk'}(\rho') \equiv 0 \) for \( k \neq k' \). The inherent field of rod \( k \) is given by (3). The current vector
averaged over the boundary of the selected rod as arising from the other rods is zero, which
is not obvious but which, however, does not mean that the resultant field is not dependent on
the number of rods in the central zone. A rod makes a nonzero contribution to the field aver-
gaged over the boundary of the central zone, and thus senses the presence of the other rods
not directly but via the boundary of the central zone. The mean field from rod \( k' \) written
for the boundary of the central zone is

\[
\bar{\mathbf{j}}_{kk'}(R) = \bar{\mathbf{K}}_{kk'}(R) \mathbf{A},
\]

where

\[
\bar{\mathbf{K}}_{kk'}(R) = \begin{pmatrix} \frac{4}{R} & 0 & 0 \\ 0 & \frac{1}{R} & 0 \\ \sqrt{\frac{14}{15}} \frac{1}{R} \left[ 1 - 2 \left( \frac{Z}{R} \right)^2 \right] - \frac{1}{\sqrt{\frac{15}{R}}} \frac{1}{R} \left[ 1 - 2 \left( \frac{Z}{R} \right)^2 \right] \frac{1}{R} \end{pmatrix};
\]

where \( Z \) is the distance between the center of the cluster and that of rod \( k' \), while \( R \) is the
radius of the central zone.