Fluid models for relativistic electron beams

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A system of equations is provided that may be used in the study of relativistic charged particle beams. The equations are based upon the equations of the kinetic theory for first, second and third order moments and the system is closed by letting the third order moment depend on the lower order ones. The form of that dependence is formally equal to the explicit constitutive function given by extended thermodynamics. However, here the contributions to the third order moment can be classed as being different in order of magnitude, because there is a smallness parameter characterizing the small dispersion of the particle beam. The resulting system of equations is quite specific, it is fully covariant and it is equivalent to a symmetric hyperbolic system thus ensuring existence and uniqueness of solutions.

1 Introduction

Intense relativistic electron beams are of great interest in several areas of astrophysics, such as solar flares [1] and pulsar magnetospheres [2] and in laboratory plasma physics, e.g. for free electron lasers in thermonuclear fusion [3].

In this field calculations for specific experimental situations are performed by using numerical simulation techniques, e.g. particle pushing solution schemes for the Vlasov equation. Numerical simulation techniques, however, have some limitations. Apart from being time-consuming, they are also ill-suited for treating problems of a general nature like the stability of equilibrium configurations.

In many problems one is interested in questions of general nature whose answers can provide clues as to the physical mechanisms which are responsible for a class of phenomena. For these problems an analytical or semi-analytical approach is better suited. For the system under consideration here, which is a relativistic charged particle beam, an adequate description is in terms of a one-particle distribution function obeying the relativistic Vlasov or, depending on circumstances, the Fokker-Plank equation.
Such a kinetic equation is very difficult to study analytically unless one limits oneself to linear stability analysis around a specific equilibrium distribution function. However, for many purposes, one would like to study the non-linear stability of a given macroscopic equilibrium configuration, which corresponds to a broad range of microscopic states described by distribution functions. For this purpose it is convenient to resort to a reduced description in terms of an appropriate fluid model.

Several fluid models for relativistic electron beams have been proposed starting from the simple cold fluid one corresponding to zero temperature [4]. Finite temperature fluid models have been developed, among others, by Toepfer [5] who assumes that the electrons have a thermal distribution. A more general approach is based on the hierarchy of moment equations and on truncating by equating to zero the 3rd order off-center moment. Such an assumption is justified for a beam or a warm plasma in general, for which as consequence of the small dispersion in momentum space, the off-center moments of increasing order are expected to be of decreasing order of magnitude.

Along these lines Siambis [6] and Newcomb [7] have proposed relativistic fluid models with so-called generalized state equations. However these models are not manifestly covariant and this is unsatisfactory in a relativistic theory. A relativistic covariant formulation has been obtained by Amendt and Weitzner [8, 9] and a detailed study of the linear wave modes in this model has been made by Amendt [10].

All these models have the drawback that some of the algebraic constraints on the moments, which arise from the requirement that the moments derive from a distribution function, are satisfied only approximately as a consequence of having equated to zero the off-center 3rd order moment. An attempt by Amendt & Weitzner [8] to satisfy the constraints exactly by truncating differently (i.e. by equating the off-center 3rd order moment to a non-vanishing suitable combination of lower order moments) led to a resulting system of partial differential equations of elliptic type, which is definitely unsuited for describing evolution phenomena. The aim of this article is to propose a truncation procedure such that the constraints are satisfied exactly and that the resulting system of partial differential equations is hyperbolic under suitable conditions. In fact, as a bonus of our methods, we shall find that our system is equivalent to a symmetric hyperbolic one, which is very satisfactory from the mathematical viewpoint, e.g. for general existence and uniqueness results [11].

The truncation method we employ assumes that the 3rd order off-center moment $S^{\text{off}}$ can be expressed as a function of the previous moments, viz. particle density, average velocity and stress tensor, satisfying the constraints exactly. As a further restriction we impose that there exists a supplementary conservation law and that such a conservation law holds as a consequence of the truncated moment equations, with the above mentioned truncation. The physical motivation for this additional requirement is that from the kinetic equation one can obtain an "entropy" balance according to which the divergence of the "entropy" flux four-vector $h$ is equal to a source.

Stated in this form this problem is exactly that of relativistic extended thermodynamics of Liu, Müller and Ruggeri [12]. Therefore we can utilize the (ap-