Moment-curvature relations for a pseudoelastic beam

T. Atanacković and M. Achenbach

Pseudoelastic bodies have very simple stress-strain diagrams for uniaxial tensile and compressive loading. In particular, yield and recovery occur at fixed stresses. And yet, the moment-curvature diagrams for bending and unbending of a beam are fairly complex, because the stress and strain fields are non-uniform. The paper shows stress profiles within the beam for pure bending and arrives at explicit equations for loading and unloading curves.

1 Introduction

In some temperature range shape memory alloys exhibit pseudoelasticity. In that range the stress-strain curve of a single crystal under tension and compression has the form shown in Fig. 1. There is a yield limit and a recovery limit so that in a loading–unloading experiment the state of the body runs through

Fig. 1. Stress-strain diagrams for a pseudoelastic body
Fig. 2. Dimensions of a beam segment. Coordinates. $r_0$ is the radius of curvature

a hysteresis loop. At a higher temperature the hysteresis loops are farther away from the origin and altogether smaller.

Given a stress-strain diagram of the type shown in Fig. 1 we shall in this paper derive a moment curvature relation for a pseudoelastic beam in bending. As a result of the non-uniformity of the stress- and strain-fields in the beam this relation will be considerably more complex than the $(a, e)$-relation. We note that in [1] the moment-curvature diagram was assumed to have the same general characteristics as the $(a, e)$-diagrams of Fig. 1. The cross section of the beam is assumed to be rectangular. Its dimensions are shown in Fig. 2 along with the choice of coordinates for the subsequent analysis.

2 Pure bending and unbending of a rectangular beam

We redraw a $(a, e)$-diagram for a memory alloy in the pseudoeelastic range in Fig. 3 and introduce some notation that will frequently be referred to in the sequel.

The yield in loading starts at the point $(a_2, e_2)$ and ends at $(a_2, e_4)$. In unloading from a strain $e > e_4$ and yield starts at $(a_1, e_3)$ and ends at $(a_1, e_1)$. If the unloading starts from $e_2 < e < e_4$ (say from point $R$) it proceeds along the line $RR'$ then $R'S$ and finally $S0$. The lines $0B$ and $RR'$ are assumed to be parallel.