Moment-curvature relations for a pseudoelastic beam

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Pseudoelastic bodies have very simple stress-strain diagrams for uniaxial tensile and compressive loading. In particular, yield and recovery occur at fixed stresses. And yet, the moment-curvature diagrams for bending and unbending of a beam are fairly complex, because the stress and strain fields are non-uniform. The paper shows stress profiles within the beam for pure bending and arrives at explicit equations for loading and unloading curves.

1 Introduction

In some temperature range shape memory alloys exhibit pseudoelasticity. In that range the stress-strain curve of a single crystal under tension and compression has the form shown in Fig. 1. There is a yield limit and a recovery limit so that in a loading—unloading experiment the state of the body runs through

![Stress-strain diagrams for a pseudoelastic body](image)

Fig. 1. Stress-strain diagrams for a pseudoelastic body
Fig. 2. Dimensions of a beam segment. Coordinates. $r_0$ is the radius of curvature

Given a stress-strain diagram of the type shown in Fig. 1 we shall in this paper derive a moment curvature relation for a pseudoelastic beam in bending. As a result of the non-uniformity of the stress- and strain-fields in the beam this relation will be considerably more complex than the $(\sigma, \varepsilon)$-relation. We note that in [1] the moment-curvature diagram was assumed to have the same general characteristics as the $(\sigma, \varepsilon)$-diagrams of Fig. 1. The cross section of the beam is assumed to be rectangular. Its dimensions are shown in Fig. 2 along with the choice of coordinates for the subsequent analysis.

2 Pure bending and unbending of a rectangular beam

We redraw a $(\sigma, \varepsilon)$-diagram for a memory alloy in the pseudoelastic range in Fig. 3 and introduce some notation that will frequently be referred to in the sequel.

The yield in loading starts at the point $(\sigma_2, \varepsilon_2)$ and ends at $(\sigma_2, \varepsilon_4)$. In unloading from a strain $\varepsilon > \varepsilon_4$ and yield starts at $(\sigma_1, \varepsilon_3)$ and ends at $(\sigma_1, \varepsilon_1)$. If the unloading starts from $\varepsilon_2 < \varepsilon < \varepsilon_4$ (say from point $R$) it proceeds along the line $RR^I$ then $R^I S$ and finally $SO$. The lines $0B$ and $RR^II$ are assumed to be parallel

Fig. 3. $(\sigma, \varepsilon)$-diagram with characteristic points