Moment-curvature relations for a pseudoelastic beam

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Pseudoelastic bodies have very simple stress-strain diagrams for uniaxial tensile and compressive loading. In particular, yield and recovery occur at fixed stresses. And yet, the moment-curvature diagrams for bending and unbending of a beam are fairly complex, because the stress and strain fields are non-uniform. The paper shows stress profiles within the beam for pure bending and arrives at explicit equations for loading and unloading curves.

1 Introduction

In some temperature range shape memory alloys exhibit pseudoelasticity. In that range the stress-strain curve of a single crystal under tension and compression has the form shown in Fig. 1. There is a yield limit and a recovery limit so that in a loading-unloading experiment the state of the body runs through

Fig. 1. Stress-strain diagrams for a pseudoelastic body
Given a stress-strain diagram of the type shown in Fig. 1 we shall in this paper derive a moment curvature relation for a pseudoelastic beam in bending. As a result of the non-uniformity of the stress- and strain-fields in the beam this relation will be considerably more complex than the \((\sigma, \varepsilon)\)-relation. We note that in [1] the moment-curvature diagram was assumed to have the same general characteristics as the \((\sigma, \varepsilon)\)-diagrams of Fig. 1. The cross section of the beam is assumed to be rectangular. Its dimensions are shown in Fig. 2 along with the choice of coordinates for the subsequent analysis.

2 Pure bending and unbending of a rectangular beam

We redraw a \((\sigma, \varepsilon)\)-diagram for a memory alloy in the pseudoelastic range in Fig. 3 and introduce some notation that will frequently be referred to in the sequel.

The yield in loading starts at the point \((\sigma_2, \varepsilon_2)\) and ends at \((\sigma_2, \varepsilon_4)\). In unloading from a strain \(\varepsilon > \varepsilon_4\) and yield starts at \((\sigma_1, \varepsilon_3)\) and ends at \((\sigma_1, \varepsilon_1)\). If the unloading starts from \(\varepsilon_2 < \varepsilon < \varepsilon_4\) (say from point \(R\)) it proceeds along the line \(\overline{RR}^\text{II}\) then \(\overline{RR}^\text{II}S\) and finally \(50\). The lines \(0B\) and \(\overline{RR}^\text{II}\) are assumed to be parallel.

![Fig. 3. \((\sigma, \varepsilon)\)-diagram with characteristic points](image-url)