ON THE CLASSICAL THERMAL CONDUCTIVITY IN A TOROIDAL PLASMA

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In toroidal equilibrium configurations the drift flow of heat causes a redistribution in plasma temperature along the magnetic surfaces. The resulting temperature gradient is equalized by an axial "nonmagnetic" heat flow, which leads to a certain effective flow of heat across the magnetic surfaces. This additional toroidal heat flow proves to be greater than the magnetic flow, which determines the heat loss from a plasma in a cylindrical geometry. The effective toroidal coefficient of thermal conductivity is calculated for "smooth" toroidal systems of Tokamak and stellarator types, the latter having a spatial figure-eight form. Expressions are also obtained for the distribution of the electric potential associated with the above-mentioned temperature redistribution.

It is well known that the idea of thermally insulating a plasma by means of a magnetic field depends on the fact that the coefficient of thermal conductivity depends strongly on magnetic field intensity. When charges collide in a magnetic field with straight lines of force they can be displaced across the lines of force only by a distance of the order of the mean Larmor radius \( r_B = \frac{|eB|}{Mv} \). It follows from this that the coefficient of thermal conductivity in a transverse direction is

\[
\kappa_\perp \sim \frac{n r_B^2}{\tau},
\]

where \( \tau \) is the average time between collisions of ions of given type. The manner in which the charges move in a toroidal geometry is complex. In addition to the main toroidal field, the additional magnetic field required to balance toroidal drift causes the charges to drift in the toroid cross section around circles whose center is shifted from the center of the cross section of the magnetic surface by an amount \( \delta \) depending on the axial and transverse velocities. It was pointed out in [1] that when these drift trajectories for charges with different velocities are "mixed up", a further flow of heat must take place along the line of centers of the drift circles. The corresponding coefficient of thermal conductivity is

\[
\kappa_\perp \sim n \frac{\delta^2}{\tau}.
\]

If the drift in a toroidal (axial) magnetic field \( B_\parallel \) is balanced by means of an axial current, producing an azimuthal field \( B_\omega \) (Tokamak system), then according to [1]

\[
\delta = \frac{[2v_\parallel^2 + v_\perp^2]}{2v_\parallel^2} \cdot \frac{Mc}{eB_\omega} = \frac{QB}{RB_\omega},
\]

where \( R \) is the toroid radius; \( \rho \) is the distance from the center of the magnetic surface cross section; \( v_\parallel \) and \( v_\perp \) are the axial and transverse components of charge velocity. It is clear from this that the thermal conductivity of a plasma in a toroidal geometry is about \( 1 + \frac{\rho^2 B_\omega^2}{R^2 B_\parallel^2} \) times greater than in a cylinder, and is determined not by the main magnetic field but by the auxiliary \( B_\omega \) (if it is not too strong).
The above formula for the coefficient of thermal conductivity was deduced without taking account of the electric fields ever present in a plasma, and consequently there is some doubt about its validity. Scant attention has been given in recent years to providing more accurate estimates for the coefficient of thermal conductivity, most work having been concerned with plasma instabilities. The lack of a satisfactory theoretical formula for the coefficient of heat conductivity has recently been pointed out once more by L. A. Artsimovich in connection with a comparison of theory based on the "classical" nonturbulent transfer equations with experimental data from Tokamaks. The question of thermal conductivity is clearly of prime importance when appraising the merits of toroidal systems for containing high-temperature plasmas.

In the present paper the conductivity in a toroidal geometry is worked out on the basis of a macroscopic description of the plasma utilizing the classical transfer equations. With the macroscopic approach the increased thermal conductivity is due to the drift and axial "nonmagnetic" flows of heat. The drift flow creates a certain mean temperature gradient in the direction of the binormal to the magnetic axis (in exactly the same manner as charge drift leads to the polarization of the plasma in the same direction). The surfaces $T = \text{const}$ intersect the magnetic lines of force as shown in the figure. It can be seen from the figure that the axial temperature gradient of the nonmagnetic heat flow along the magnetic lines of force gives rise in effect to heat transfer in a direction across the isothermals. The expression for the heat flow obtained from the hydrodynamical transfer equations agrees with Budker's estimate based on a microscopic picture of charge motion.

In the present paper the heat flow in a toroidal plasma is worked out both for Tokamak geometry and stellarator figure-eight geometry.

The appearance of a temperature gradient due to a drift flow of heat gives rise to a certain additional polarization of the plasma column; this is also taken into account in the present article.

**Heat Transfer in Toroidal Geometry**

The temperature gradient causes a heat flow which is carried by the charged particles of the plasma. The plasma is situated in a strong magnetic field, and the heat flow consists of a transverse magnetic flow $q_L$, a drift flow $q_A$ and a normal flow $q||$ [2]:

$$q = q_L + q_A + q|| = -\chi_\perp \nabla \cdot T + \frac{5}{2} \frac{\alpha nT}{\tau} [\mathbf{B} \cdot \nabla T] - \chi_\parallel \nabla \cdot T,$$

where

$$\chi_\perp = \alpha_\perp \frac{nT \tau}{M} ; \quad \chi_\parallel = \alpha_\parallel \frac{nT \tau}{M} ;$$

$M$ and $\alpha$ are the mass and charge of the particles under consideration (for electrons $z = -1$); $n$, $T$ are their density and temperature; $\tau$ is the mean time between collisions; $\omega = \frac{eB}{M}c$, is the cyclotron frequency; $\alpha_\perp$ and $\alpha_\parallel$ are numerical coefficients which have the following values for a hydrogen-type plasma: for ions $\alpha_\perp = 2, \alpha_\parallel = 3.9$, for electrons $\alpha_\perp = 4.66, \alpha_\parallel = 3.16$.

For $\omega \tau \gg 1$ the above three flows differ greatly in magnitude:

$$q|| > q_A > q_L \approx 1 \frac{1}{\omega \tau} \frac{1}{\omega^2} \cdot$$

However, from symmetry considerations, the axial flow in a cylinder is zero: $q|| = 0$. The drift flow in a cylindrical plasma column is unequal to zero, although from the symmetry of the geometry its divergence must identically equal zero; heat transfer can thus only take place due to magnetic flow $q_L$.

To estimate the thermal conductivity in a toroidal geometry we expand the solution in terms of the small parameter $1/\omega \tau$.

1. In the zero-th approximation we drop all terms.