SIMULATION OF HEAT AND MASS EXCHANGE
WITH NATURAL CONVECTION

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Recently the interest in natural circulation has intensified [1]. Since the possibilities of an experimental investigation of such regimes with operating nuclear reactors are limited, various computational methods of investigation have been widely developed [2, 3].

It is necessary in connection with the solution of such problems as, for example, reactor cooling to make a nonsteady multidimensional calculation of the velocity and temperature fields in a region with uniform hydraulic and thermophysical properties; such a calculation is complicated by the significant nonuniformity of the calculated region and the inertia of natural convection processes [4]. Due to the appreciable thermal inertia of the system, it is necessary to integrate the equations of hydrodynamics and heat exchange with a sufficiently large step in the time, which frequently leads to the appearance of instability.

We shall make use of a model of a porous body [5] to describe the heat exchange processes in a reactor cooled by a single-phase compressible liquid. Then one can write the initial system of equations which express the laws of conservation of mass, momentum, and energy in the form

\[ \nabla (\varepsilon W) = 0 \]  
\[ \frac{\partial W}{\partial t} + \nabla \times [W \times \nabla W] = -\frac{1}{\rho_0} \nabla B - \tilde{L}W + \frac{\rho(\tau)}{\rho_0} g; \]  
\[ \frac{\partial T}{\partial t} + (\rho c) \nabla T = q_v + \nabla \cdot (\lambda \nabla T), \]

where \( W \) is the velocity vector of the coolant; \( \tau \), time; \( \rho \), density; \( B \), pressure \( (B = P + \varepsilon \omega_0^2) \); \( g \) is the free-fall acceleration; \( c \), specific heat; \( c_0 \), average specific heat of a unit volume; \( q_v \), volume energy release; \( T \), temperature; subscript "c," parameter refers to the coolant; and subscript "0," value of the parameter is for some average temperature.

The porosity of the medium for the coolant, \( 0 \leq \varepsilon \leq 1 \), is taken into account in the continuity (1) and heat transfer (3) equations, and the anisotropic thermal conductivity \( \lambda \) is taken into account in Eq. (3). In contrast to the equations of motion of an ideal liquid, an additional term (the components of the tensor \( \tilde{L} \) depend on the modulus of the velocity \(|W|\)) which describes the interaction of the coolant flow with the fixed structural elements homogenized within a given zone of the calculated region is contained on the right-hand side of the expression (2). The drag coefficients in each zone are determined from experiment [6] or have been estimated from [7].

Initial distributions are specified for all the functions being sought except the pressure \( B \). Either the temperature of the liquid flowing under or the thermal flux is specified as the boundary conditions for the heat transfer equation. Specification of mixed boundary conditions is possible for the "hydrodynamic" part of the problem when the velocity vector of the liquid flowing under is given on some part of the boundary and the pressure is given on the remaining part.

An iterative type of numerical method [8] has been used to solve the problem. We shall consider its implementation by the example of a cylindrical region \( R_1 \leq r \leq R_2 \) and \( 0 \leq Z \leq H \). The region is covered by a difference grid with steps \( \Delta r_{k+1/2}, \Delta Z_{k+1/2} \). Equations (1)-(3) are written for the longitudinal \( U \) and transverse \( V \) components of the velocity vector. Having integrated the continuity equation within the limits

\[ z_k - \frac{\Delta Z_{k-1/2}}{2} \leq Z \leq z_k + \frac{\Delta Z_{k+1/2}}{2}, \quad r_l - \frac{\Delta r_{l-1/2}}{2} \leq r \leq r_l + \frac{\Delta r_{l+1/2}}{2}, \]

we obtain a relationship which links the values of the velocity components \( U \) and \( V \) on the boundaries of the integration cell:

\[
\frac{\varepsilon_{i+1/2}V_{i+1/2, k} - \varepsilon_{i-1/2}V_{i-1/2, k} + \varepsilon_{k+1/2}U_{i, k+1/2} - \varepsilon_{k-1/2}U_{i, k-1/2}}{h^2} = 0,
\]

where

\[
\varepsilon_{i+1/2} = 0.5(e_{i+1/2, k+1/2}A_{i+1/2} + e_{i+1/2, k-1/2}A_{i-1/2}).
\]

The equations of motion for \( U \) and \( V \) are approximated by finite-difference relationships which are implicit in the time and are solved for the variables being sought. For example, we have for the vertical component of the velocity \( U \) the relationship

\[
U^{(j)}_{i, k+1/2} = -\frac{\Delta \tau U_{i, k+1/2}^{(j)} + (U_{i, k+1}^{(j)} - U_{i, k-1}^{(j)})/(\delta Z_{k+1/2})}{1 + \Delta \tau U_{i, k+1/2}^{(j)} + 2\Delta \tau V_{i, k+1/2}^{(j)}/(\Delta r_{i+1/2} + \Delta r_{i-1/2})},
\]

where \( j \) is the iteration variable,

\[
(U)^{(j)}_{i, k+1/2} = 2\Delta \tau [(V)^{(j)}_{i, k+1/2} + (\Delta r_{i+1/2} + \Delta r_{i-1/2})] U* + \Delta \tau V_{i, k+1/2}^{(j)} + V_{i, k+1/2}^{(j)} + V_{i, k+1/2}^{(j)} + V_{i, k+1/2}^{(j)} + (2\Delta Z_{k+1/2});
\]

and

\[
U* = U_{i-1, k+1/2}^{(j)} \quad \text{for} \quad V_{i, k+1/2}^{(j)} > 0
\]

\[
U* = U_{i+1, k+1/2}^{(j)} \quad \text{for} \quad V_{i, k+1/2}^{(j)} < 0.
\]

We find \( V_{i, k+1/2} \) by linear interpolation of the values of \( V_{k+1/2} \) at the points \( i-1/2 \) and \( i+1/2 \).

The expressions of the form (5) obtained for the velocity are substituted into a finite-difference analog of the continuity Eq. (4), which reduces after this to a finite-difference equation of the Poisson type in the pressure \( P_{i, k} \) and is solved by the method of variable directions. The entire iterative process consists of three successive steps and starts from the calculation of the right-hand side of the equation obtained. Then the pressure field is determined, and finally the velocity is calculated from iteration formulas of the form (5). The calculation is assumed to be concluded for a given time step when a specified accuracy is achieved in the velocity.

The heat transfer equation reduces, after integration over the very same cell as the continuity equation, to a system of linear five-point equations. This system is solved with the help of the method of variable direc-