We consider the stabilization of vertical instability of a toroidal plasma pinch using passive ring conductors. The analysis is conducted by two techniques: by numerical solution of the system of linearized MHD equations and by applying the energy principle. We investigate the dependence of plasma stability on the location and the number of conductors and examine the effect produced by closing the conductors into sections. For an INTOR plasma pinch we show that a small number $N \sim 4$ of properly placed conductors of reasonable size may suppress vertical instability.

1. INTRODUCTION

The existence of a separatrix bounding the plasma pinch of elongated cross section in a tokamak with a diverter creates conditions for the onset of axisymmetric instability. The suppression of this instability by a continuous conducting shell enclosing the plasma pinch was first investigated in [1-5]. These authors studied the effect of the ellipticity of the pinch cross section, the effect of the toroidal configuration of the pinch, the effect of the pinch asymmetry in $z$, and the effect of the proximity of the wall to the plasma. Stability of vertical displacements was shown to be weakly dependent on the current distribution across the pinch [2]. An experiment conducted on the T-12 tokamak determined the critical distance $L$ between the plasma pinch and the wall that ensures the system remains stable. The width of the diverter layer $\lambda$ was found to be connected with $L$ by the relationship $0 < L - \lambda \leq 0.1a$, where $a$ is the semiminor axis of the pinch cross section [6].

The existence and operation of a diverter is determined by the obvious inequality $L > \lambda$. The presence of gaps in the stabilizing shell in the poloidal direction or replacement of the continuous shell with discrete elements may reduce $L$ and lead to breakdown of this inequality. Cuts in the toroidal direction may further reduce the value of $L$. In this context, it is relevant to investigate the stabilizing properties of a system that consists of distinct, toroidally separated sections.

In this paper we investigate the effect of the location and the number of stabilizing conductors on the stability of an INTOR plasma pinch. Systems of conductors ensuring plasma stability are proposed. The effect produced by closing the conductors into sections is investigated. We apply two approaches to plasma stability analysis: numerical solution of the system of two-dimensional linearized MHD equations and the energy principle. In the latter case, we investigate the variation of the plasma potential energy for a fixed displacement from the equilibrium position. The "fixed shift" model is compared with a more complex time-dependent model.

2. STATEMENT OF THE PROBLEM

We assume that an ideally conducting toroidal plasma with an elongated cross section $\Omega_i$ and boundary $\Gamma_p$ is surrounded by a vacuum region $\Omega_e$ containing passive toroidal conductors of cross section $S_i$, $i = 1, \ldots, M$. The skin time of the passive conductors is approximately $\tau_s \sim 5 \cdot 10^{-2}$ sec and the characteristic time of Alfvén oscillations of the poloidal field is approximately $\tau_{H_p} \sim 10^{-6}$ sec. We may therefore assume that the passive conductors do not affect the establishment of equilibrium and at the same time are ideally conducting in relation to fast MHD displacements.
1. Equilibrium

The equilibrium was found by solving the problem for the poloidal magnetic field flux function $\psi_0$:

\[
\Delta^* \psi_0 = \begin{cases} 
-J_0 j_0 (r, \psi_0 - \psi_p), & \psi_0 > \psi_p, \text{(inside the plasma)}, \\
- \sum_{i=1}^{k} j_i \delta (r - r_i, z - z_i), & \psi_0 < \psi_p, \text{(outside the plasma)}. 
\end{cases}
\]

\[\psi_0 |_{t=0} = 0; \quad \psi_0 (\infty) = 0. \tag{2}\]

Here $J_0$ is the total plasma current, $k$ is the number of external control conductors, $I_i$, $r_i$, $z_i$ are the magnitudes and the coordinates of the external control currents, $j_{p0}$ is the distribution function of the plasma current over the magnetic surfaces. The solution method for problem (1)-(2) is described in [5, 8]. The equilibrium was calculated for INTOR parameters [7].

2. Time-Dependent Problem

We use the equations of ideal magnetohydrodynamics of an incompressible fluid,

\[
\frac{\partial H_{i}}{\partial t} = \text{rot} [\text{V} H_{o}],
\]

\[
\rho_0 \frac{\partial \text{V}}{\partial t} = -\nabla \rho + F, \quad F = (H_{o} \text{V}) H_{i} + (H_{i} \text{V}) H_{o}, \quad \Delta \rho = \text{div} F. \tag{3}\]

Here $V$ is the velocity, $H_{oi}$ is the equilibrium field inside the plasma, $H_i$ is the field perturbation, $p$ is the perturbation of the generalized pressure $p_0 + H_0 \frac{\rho}{2}$, $\rho_0 = \text{const}$ is the plasma density.

In the vacuum region, we have Maxwell's equations

\[
\text{div} H_e = 0; \quad \text{rot} H_e = 0. \tag{4}\]

Here $H_e$ is the perturbation of the field outside the plasma. The system of equations (4) can be reduced to one equation for the perturbation flux function $\psi_e$:

\[
\Delta^* \psi_e = r \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi_e}{\partial r} \right) + \frac{\partial^2 \psi_e}{\partial z^2} = 0. \tag{5}\]

Here $r, \varphi, z$ is the system of cylindrical coordinates with the axis $z$ directed along the major axis of the torus.

At the plasma—vacuum boundary $\Gamma_p$, we have the condition of continuity of the normal magnetic field component and the pressure balance condition,

\[
\psi_e |_{r_p} = \psi_l |_{r_p},
\]

\[
\rho |_{r_p} = \frac{1}{r} H_0 u_0 \frac{\partial \psi_e}{\partial n}. \tag{7}\]

Here $\psi_l$ is the magnetic flux perturbation inside the plasma. Now, the flux function $\psi_e$ satisfies the following regularity conditions:

\[
\psi_e (0, z) = 0; \quad \psi_e (\infty) = 0. \tag{8}\]

We considered two models of ideally conducting stabilizing conductors:

I. Mutually insulated conductors, continuous along the large circumference of the torus.

II. Conductors closed to form sections in the toroidal direction, with negligibly small clearances between sections. In model I, the boundary conditions on the passive conductors were formulated as follows:

\[
\psi_e |_{r_k} = 0, \quad k = 1, \ldots, M, \tag{9}\]

which corresponds to constant magnetic flux through the surface surrounded by a closed ideally conducting conductor.

Let us describe model II. In this case, the following system of boundary conditions is specified on the conductor surfaces: