AN AXIOMATIC BASIS FOR DISTRIBUTIONAL EQUALITY IN UTILITARIANISM

ABSTRACT. An axiomatic basis for a social preference ordering with interval-scaled utility levels satisfying the principles of anonymity and pareto superiority is elaborated. The ordering is required to be sensitive to distributional equality: Redistribution of utility income from poor to rich persons without changing their social rank should lead to a superior evaluation. The axiom of separability is weakened in order to make it compatible with distributional equality. We prove that every continuous ordering satisfying the upper axioms can be represented by a utility function which is positively linear on the convex cone of rank-ordered utility vectors. A modified unnormalized Gini coefficient is one possible choice, but it contradicts, as well as related proposals, the principle of adequacy of means for some distribution problems.

1. GENERAL PRELIMINARIES

In [8] a novel conception of utilitarianism is developed. Basically it consists of a single decision principle represented by a utility function of the vector of each person’s subjective utility value. The function can be written as a linear combination of total utility sum and some linear measure of dispersion. It is unique up to a global parameter which could be interpreted as the degree of justice or sensitivity to distributional equality. The axiomatisation provided there has the disadvantage of explicitly referring to a special measure of distributional equality. It is the aim of the paper to give an intuitively motivated axiomatic basis leading to a generalized concept, where each social rank is associated with its own parameter of justice.

Let $\succeq$ be a single binary relation on the $n$-dimensional euclidean vector space, where each component of a vector represents the person’s interpersonally standardized utility level. The following argumentation does not explicitly refer to the way the utility value is defined and is therefore open to a broad variety of interpretations. For example, the raw utility value representing the person’s subjective state is often modified according to his desert or merit. Those principles of moral rightness are discussed elsewhere.\textsuperscript{1} In any case, the decision principle

is applicable to the pure distribution problem, where all persons have equal ethical merit.

NOTATIONS AND DEFINITIONS. \( \mathbb{R}^n \) is the \( n \)-dimensional euclidean space, \( x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \) is a utility vector, and \( x_i \) the \( i \)th person's utility level. \( e = (1, 1, \ldots, 1) \) is called the one-vector. A permutation operator \( P \) is a linear operator, which turns each vector into a permutation \( P(x_1, x_2, \ldots, x_n) = (x_{j_1}, x_{j_2}, \ldots, x_{j_n}) \) with \( j(.) \) bijective. The non-linear standard permutation \( \bar{x} \) transforms each vector to ascending component order:

\[
(1) \quad \bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \quad \text{with} \quad \bar{x}_1 \leq \bar{x}_2 \leq \cdots \leq \bar{x}_n.
\]

Observe that for each \( \alpha \geq 0 \) and each \( \beta \)

\[
(2) \quad y = \alpha x + \beta e \rightarrow \bar{y} = \alpha \bar{x} + \beta e,
\]

\[
(3) \quad \bar{x} + \bar{y} = \bar{x} + \bar{y}.
\]

If \( \geq \) is a relation, let \( x \sim y \) denote \( x \geq y \land y \geq x \) and let \( x \succ y \) stand for \( x \geq y \land \neg x \leq y \). \( \geq \) is antisymmetrical modulo permutation, if \( \forall xy \ x \geq y \land y \geq x \rightarrow \bar{x} = \bar{y} \).

The following three axioms define the social preference ordering to be a complete ordering, sensitive only to the distribution of utility values and not to the persons who have them, and involving interval-scaled utility levels.

ORDER AXIOM. \( \geq \) is a reflexive, transitive and connex relation on \( \mathbb{R}^n \).

AXIOM OF SCALE INVARIANCE. \( \geq \) is invariant under a common positive linear transformation of the utility values:

\[
(4) \quad x \geq y \iff \alpha x + \beta e \geq \alpha y + \beta e \quad \text{for each} \quad \alpha \geq 0 \quad \text{and} \quad \beta
\]

AXIOM OF ANONYMITY. For each permutation operator \( P \)

\[
(5) \quad x \geq y \iff Px \geq Py
\]

LEMMA 1. For a connex ordering \( \geq \) the axiom of anonymity is equivalent to