EQUIVALENCE CLASSES OF PROBLEMS

To the set of all vectors \( \{ f_m \} \) corresponds the set of families of \( E_k(\mathbb{R}^2) \) location problems. The number of problem families is estimated at 884,736.

If some metric \( p* \) is defined on the set of vectors, then the pair \( (f_m, p*) \) forms a metric space, which corresponds to location problems from the set \( \{ W* \} \). A point \( f_m \) in this space is a representative of a family of problems.

On the set of problem families \( \{ f_m \} \) we define equivalence classes by the following attribute. The representatives \( f_m^1 \), \( f_m^2 \) of problem families belong to the same equivalence class if they correspond to the same vector \( V_m^i \), \( i = 1, 2, \ldots, 2^{m+15} \).

With a suitable metric \( p* \), we can compare different equivalence classes of problems and thus develop a more flexible approach to the selection of solution methods. The choice of solution methods based on the values of the vector \( v \) is a strategic decision for the entire equivalence class.

Thus, when constructing the correspondence \( f_{wp*} : \{ W* \} \Rightarrow \{ P* \} \), we have an objective sequence of mappings \( f_{wp*} : \{ W* \} = \{ f_m* \} = \{ V \} = \{ P* \} \). The element \( p* \in \{ P* \} \) can be chosen based on fixed values of the elements of the vector \( V \) by three techniques: (i) in one pass, when the choice of the method \( p* \in \{ P_1, \ldots, P_c \} \subseteq \{ P* \} \) is obvious, (ii) with the construction of \( V^i \), \( i \in \{ 1, \ldots, K_m \} \), or finally (iii) with the construction of a particular realization \( m* \) of the mathematical model of class \( m_r \), \( i \in \{ 1, \ldots, K_m \} \).

We assume that the system is open, i.e., it allows introduction of new elements associated with the appearance of new additional constraints, new spatial shapes of objects and location regions, and more efficient methods for the solution of \( E_k(\mathbb{R}^2) \) location problems [6].

LITERATURE CITED


INTEGRAL MODEL OF A FUZZY SYSTEM

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An integral model of a fuzzy system is constructed, combining the properties of both inductive and deductive models.

The management of complex organizations usually involves semantic-logical and computational-logical problems that are difficult to formalize. Such poorly formalizable problems can be solved only on the basis of expert judgments, which are formulated in verbal form, i.e., on a qualitative level. Expert knowledge is formalized using conditional statements of the form "If \( X \) then \( Y \)," where \( X \) and \( Y \) are fuzzy sets in the input universe \( U \) and the output universe \( V \), respectively.

The sets X and Y are interpreted as the fuzzy input and the fuzzy output of some system, and the relationship between them is determined by the fuzzy system model R. The set of pairs \((X^{(i)}, Y^{(i)})\), \(i = 1, \ldots, K\), is defined as a specification of the system and the input-output relation \(R^{(i)}\) is defined as a fuzzy granule. The specification of the system provides initial data for the construction of the model. If context-dependent labels in a natural language are associated to the fuzzy sets defining the system input and output, we have a linguistic specification.

The fuzzy granules over given input-output pairs and the corresponding fuzzy system model can be constructed in two ways: using a two-sided solution that does not allow for the causal relationship between input and output [1] and using solutions that allow for the input-output causality [2].

In this paper, we study an integral fuzzy system model in the context of duality: an inductive model that incorporates a causal input-output relationship according to which a "larger" input set produces a "larger" output set and a deductive model which incorporates a causal input-output relationship according to which a "larger" input set produces a "smaller" output set. Our problem is to construct a consistent integral fuzzy system model R which combines both inductive and deductive properties.

**REPRESENTATION OF THE SYSTEM USING A FUZZY CONDITIONAL STATEMENT**

To the conditional statement "If X then Y else Z" \((X \rightarrow Y(Z))\) corresponds a representation of the fuzzy system in the form [1]

\[
R = X \times Y \cup \bar{X} \times Z. \tag{1}
\]

Then the fuzzy system model may be written in a compact form as

\[
Y^{(i)} = X^{(i)} \circ R, \tag{2}
\]

where \(\circ\) is the (max-min) operator, and \(R = \bigcup_i R^{(i)}\), or in equivalent form

\[
\mu_Y(i)(v) = \sup_{u \in U} \min \{\mu_X(i)(u), \mu_R(u, v)\}. \tag{3}
\]

The solution of (2) is usually taken in the form of the direct product \(\bigcup_i X^{(i)} \times Y^{(i)}:\)

\[
\mu_X(i) \times Y^{(i)}(u, v) = \min \{\mu_X(i)(u), \mu_Y(i)(v)\}, \quad \forall u \in U, \quad \forall v \in V. \tag{4}
\]

The solution (3) is a two-sided solution that ignores the causal relationship between input and output. The representation of \((X^{(i)}, Y^{(i)})\) in terms of \(X^{(i)} \times Y^{(i)}\) only assumes the possibility of simultaneous occurrence of \(X^{(i)}\) and \(Y^{(i)}\). The separate fuzzy granules must be compatible and aggregated into the fuzzy system model.

Note that the fuzzy system (1) defines an inexact representation of the conditional statement \(X \rightarrow Y(Z)\). However, for the case of classical representation of input and output sets, the model (1) corresponds to an exact representation of the system.

Starting from this point, we state a proposition concerning the representation of the fuzzy system R. We first define the fuzzy conditional statement \(X \rightarrow Y(Z)\) in terms of level sets, i.e., in the form

\[
R = \sup_{\beta \in [0, 1]} \beta \left( \bigcap_{a \leq \beta} R^a \right), \tag{5}
\]

where \(R^a = X^a \times Y^a \cup \bar{X}^a \times Z^a\), and assume that the fuzzy sets X and Y are normal, i.e.,

\[
\max_{u \in U} \mu_X(u) = \max_{v \in V} \mu_Y(v) = 1 \quad \text{and} \quad \min_{u \in U} \mu_X(u) = \min_{v \in V} \mu_Y(v) = 0.
\]

Then we have the following proposition.

**Proposition.** The representation of the system \(R = \sup_{\beta \in [0, 1]} \beta \left( \bigcap_{a \leq \beta} R^a \right)\), where \(R^a = X^a \times Y^a \cup \bar{X}^a \times Z^a\), is equivalent to the representation \(R = X \otimes Y\) with the membership function

\[
\mu_R(u, v) = \begin{cases} \mu_X(v) \lor \mu_Y(u), & \text{if} \quad \mu_X(u) \leq \mu_Y(v), \\ \mu_X(u) \land \mu_Y(v), & \text{if} \quad \mu_X(u) > \mu_Y(v). \end{cases}
\]

The proof is similar to that in [2].