A METHOD FOR CALCULATION OF A HIGH-FREQUENCY DISCHARGE

I. S. Gushchin

1. Introduction

There has been a recent increase of interest in the study of discharges produced by high-frequency fields [1]. This topic gives rise to problems which may be viewed as a new, little studied object in the theory of dissipative structures [2]. One of these problems can be formulated in the following way. In an unbounded two-dimensional domain of the variables $x$, $y$, solve the system of equations

$$\frac{\partial u}{\partial t} = D\Delta u + (|\vec{E}|^{2\beta} - 1) u,$$

(1)

$$\text{div} ((1-u-i\nu u) \vec{E}) = 0.$$  

(2)

For the function $u(x, y, t)$ we have the initial distribution $u(t = 0) = u_0(x, y)$, which does not vanish only in the neighborhood of the origin. It is required to find a solution which for $(x^2 + y^2)^{1/2} \to \infty$ satisfies the conditions $u \to 0$, $\vec{E} \to E_0 \vec{T}_x$, where $\vec{T}_x$ is the unit vector in the direction of the $x$-axis, $i$ is the complex unity.

Equation (1) describes diffusion, ionization, and sticking of electrons under the action of the external high-frequency field with the amplitude $E_0$. Equation (2) is a consequence of Maxwell's equations for the electromagnetic field. The variables have the following physical meaning: $\vec{E}$ is the electrical field, $u$ is the dimensionless electron density, $\beta$ is the power exponent in the expression for the dependence of ionization frequency on the electric field strength, $\nu$ is the ratio of electron collision frequency to the field frequency, $D$ is the electron diffusion coefficient.

When solving the system (1), (2), we seek the field $\vec{E}$ in the form $\vec{E} = -\nabla \varphi$, where $\varphi = f + ig$ is the field potential. The unbounded domain of the variables $x$, $y$ is replaced with the rectangle $P = [x_{\text{min}}, x_{\text{max}}] \times [y_{\text{min}}, y_{\text{max}}]$ that contains the origin. The rectangle is taken sufficiently large so that for a given finite time interval the perturbation of the functions $u$ and $\vec{E}$ does not reach the boundary. If we restrict the analysis to symmetrical initial distributions $u_0(x, y)$, then by symmetry we need to consider only one quadrant in the $(x, y)$ plane. As a result, we obtain the problem

$$\frac{\partial u}{\partial t} = D\Delta u + (E_0^{2\beta} - 1) u,$$

(3)

$$\nabla (1-u) \nabla f + \nu \nabla g = 0,$$

(4)

$$\nabla (\nu u \nabla f - (1-u) \nabla g) = 0,$$

(5)

inside the domain $(x, y) \in P = (x_{\text{min}}, 0) \times (0, y_{\text{max}})$;

$$\frac{\partial u}{\partial n} = 0, \quad f = 0, \quad g = 0 \quad \text{on \partial x \text{ axis: } x \in [x_{\text{min}}, 0], \quad y = 0;}$$

$$\frac{\partial u}{\partial n} = 0, \quad f = 0, \quad g = 0 \quad \text{on \partial y \text{ axis: } x = 0, \quad y \in [0, y_{\text{max}}]}.$$

$$u = u_\infty, \quad f = E_0 x, \quad g = g_\infty \quad \text{on the outer boundary } x \in [x_{\text{min}}, 0], \quad y = y_{\text{max}} \text{ or } x = x_{\text{min}}, \quad y \in [0, y_{\text{max}}].$$

The initial function $u(x, y, 0) = u_0(x, y)$, $(x, y) \in P$, is localized in the neighborhood of the origin and is compatible with the boundary conditions on the coordinate axes.


1046-283X/93/0401-0095$12.50 ©1993 Plenum Publishing Corporation
The problem is nonlinear. An efficient method for solving nonlinear partial differential equations is the finite-difference method, which is indeed used in this paper. Since problems of this class may have growing solutions that are characterized by rapid variation of variables in narrow space regions, severe restrictions on the time grid increment are avoided by using schemes that lead to implicit difference equations [2]. The nonlinear equations obtained in this way are best solved by Newton’s method, because it converges rapidly and has good initial approximations in our problem. The system of nonlinear equations is solved by the direct method. The following sections consider each stage of the method in more detail.

2. The Difference Scheme

In the rectangle $\tilde{P}$ define the nonuniform grid $\omega_h = \{x_1 = x_{\text{min}}, x_k = x_{k-1} + h_{k-1}, k = 2, ..., k_{\text{max}}; y_1 = y_{\text{min}} = 0, y_l = y_{l-1} + h_{l-1}, l = 2, ..., l_{\text{max}}\}$. In the time variable $t$, we define the grid $\omega_t = \{t_0 = 0, t_m = t_{m-1} + \tau_m, m = 1, 2, ..., m_{\text{max}}\}$. The grid functions are defined at the nodes of the difference grid $\omega = \omega_h \times \omega_t$. We denote them by the same symbols as in the differential problem. Equation (3) is approximated by a weighted difference equation [3] and Eqs. (4), (5) are approximated by the scheme from [4]. As a result, we obtain the following difference equations:

1) on the left-hand outside boundary of the rectangle ($k = 1, 1 \leq l \leq l_{\text{max}}$)

$$F_k^1 = u_{k1}^n - u_{\text{in}} = 0, \quad F_k^3 = f_{k1}^n - E_{k1}^n = 0, \quad F_k^3 = g_{k1}^n - g_{\text{in}} = 0;$$

2) at the grid nodes on the $Ox$ axis ($2 \leq k \leq k_{\text{max}} - 1, 1 \leq l \leq l_{\text{max}} - 1$)

$$F_k^1 = -u_t + D \left( \frac{u_{x_{k-1}} - u_{x_k}}{h_x} \right)^{\alpha_t} + (u\Delta)^{\alpha_t}, \quad \Delta = E_{k1}^n - 1,$$

$$E_k^2 = 0.5 \left( f_x f_x + f_y f_y + g_x g_x + g_y g_y + f_{xx} f_{xx} + f_{yy} f_{yy} + g_{xx} g_{xx} + g_{yy} g_{yy} \right),$$

$$F_k^2 = \left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_x + \frac{u - u^{(-h_y)}}{2} g_x \right]_{x_k}^x +$$

$$\left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_y + \frac{u - u^{(-h_y)}}{2} g_y \right]_{y_k}^y = 0,$$

$$F_k^3 = \left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_x + \frac{u - u^{(-h_y)}}{2} g_x \right]_{x_k}^x +$$

$$+ \left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_y + \frac{u - u^{(-h_y)}}{2} g_y \right]_{y_k}^y = 0;$$

3) at the interior nodes ($2 \leq k \leq k_{\text{max}} - 1, 2 \leq l \leq l_{\text{max}} - 1$)

$$F_k^1 = -u_t + D \left( \frac{u_{x_{k-1}} + u_{x_k}}{2} \right)^{\alpha_t} + (u\Delta)^{\alpha_t}, \quad \Delta = E_{k1}^n - 1,$$

$$E_k^2 = 0.5 \left( f_x f_x + f_y f_y + g_x g_x + g_y g_y + f_{xx} f_{xx} + f_{yy} f_{yy} + g_{xx} g_{xx} + g_{yy} g_{yy} \right),$$

$$F_k^2 = \left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_x + \frac{u - u^{(-h_y)}}{2} g_x \right]_{x_k}^x +$$

$$\left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_y + \frac{u - u^{(-h_y)}}{2} g_y \right]_{y_k}^y = 0,$$

$$F_k^3 = \left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_x + \frac{u - u^{(-h_y)}}{2} g_x \right]_{x_k}^x +$$

$$+ \left[ \left( 1 - \frac{u + u^{(-h_y)}}{2} \right) f_y + \frac{u - u^{(-h_y)}}{2} g_y \right]_{y_k}^y = 0;$$

4) on the top outside boundary ($2 \leq k \leq k_{\text{max}} - 1, l = l_{\text{max}}$)

$$F_k^1 = u_{k1}^n - u_{\text{on}} = 0, \quad F_k^3 = f_{k1}^n - E_{k1}^n = 0, \quad F_k^3 = g_{k1}^n - g_{\text{on}} = 0;$$

5) at the origin ($k = k_{\text{max}}$, $l = 1$)

$$F_k^1 = -u_t + D \left( \frac{u_{x_{k-1}}}{h_x} \right)^{\alpha_t} + (u\Delta)^{\alpha_t}, \quad \Delta = E_{k1}^n - 1.$$