INVERSE PROBLEMS

THE INVERSE PROBLEM OF MAGNETOTELLURIC SOUNDING
OF THE EARTH'S INTERIOR IN THE PRESENCE OF FRACTURES

I. S. Barashkov and V. I. Dmitriev

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The paper considers the inverse problem of magnetotelluric sounding for the case of \( H \)-polarization in the presence of tectonic fractures in the Earth's interior.

In 1950, Tikhonov [1] suggested that the interior structure of the Earth could be studied by examining the dependence on the frequency \( \omega \) of the impedance \( Z(\omega) \) of the Earth's natural field. The impedance \( Z(\omega) \) is defined as the ratio of the tangential components of the electric and magnetic fields measured on the Earth's surface. A uniqueness theorem has been proved for the inverse problem of magnetotelluric sounding (MTS) with one-dimensional [2] and two-dimensional [3] distributions of electrical conductivity. The studies [4, 5] have shown that the interpretation of magnetotelluric sounding data is essentially influenced by the nonhomogeneity of the subsurface layer resting on the crystalline foundation. The interpretation of MTS data also largely depends on the presence of tectonic fractures, which are cracks in the Earth's crust formed by tectonic displacement and deformation of rocks under the action of tectonic compressive, tensile, or shearing forces. These cracks affect the interpretation of MTS data because they easily divert the electric current from the subsurface sedimentary layer to the conducting halfspace under the crystalline foundation. In this paper, we consider the joint effect of subsurface layer nonhomogeneities and tectonic fractures on the interpretation of MTS data from the Earth's interior.

Consider the two-dimensional inverse problem in case of \( H \)-polarization for the fundamental three-layer model of MTS sounding in the presence of tectonic fractures in the Earth's interior. The electrical conductivity of such a medium is described by the formula

\[
\sigma(y, z) = \begin{cases} 
\sigma_1 & \text{for } 0 < z < h(y), \\
\sigma_2 = 0 & \text{for } h(y) < z < H, \\
\sigma_3 = \infty & \text{for } z > H,
\end{cases}
\]

where the first layer describes the electrical conductivity of the sedimentary shell, where both the conductivity \( \sigma_1 \) and the thickness \( h(y) \) are known (Fig. 1). The function \( h(y) \) tends as \( |y| \to \infty \) to the limit \( h_0 = \lim_{|y| \to \infty} h(y) \), and this function is not equal to the constant \( h_0 \) for \( a < y < b \). Outside the interval \( (a, b) \) the integral conductivity \( S_1 = \sigma_1 h \) is constant. The second layer rests on an ideal conducting foundation and has zero electrical conductivity \( \sigma_2 = 0 \). The thickness of the second layer is much greater than the thickness of the first layer, \( H \gg h \). We additionally assume slow variation of the function \( S_1(y) \).

Tectonic fractures in the medium are modeled by vertical cracks along the profile at the points \( y = y_j, j = 1, 2, \ldots, N, a < y_1 < y_N < b \), with conductivities \( \sigma_j \), thicknesses \( d_j \), and resistances \( r_j = (H - h_j)/(\sigma_j d_j) \), where \( h_j = h(y_j) \).

In the inverse MTS problem, the impedance \( Z(\omega, y) \) is assumed known at some points of the profile \( y = y_m \in (a, b), m = 1, 2, \ldots, M \) on the Earth's surface for various frequencies \( \omega = \omega_p, p = 1, 2, \ldots, P \). Also known are the coordinates \( y_j \) of the tectonic fractures along the profile. It is required to find the depth \( H \) of the ideally conducting foundation.
The impedance $Z$ is affected not only by the depth $H$ to the ideally conducting foundation, but also by the resistances $r_j$ of the tectonic fractures. Since $r_j$ are unknown, the solution of the inverse problem is required to produce both the depth $H$ and the resistances $r_j$.

From Maxwell’s equations for the case of H-polarization, we obtain for $0 < z < h(y)$

$$i \omega \mu H_x = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z},$$

(1)

$$\sigma_1 E_y = \frac{\partial H_x}{\partial z},$$

(2)

$$\sigma_1 E_x = -\frac{\partial H_y}{\partial y},$$

(3)

and for $h(y) < z < H$ we have outside the fractures

$$i \omega \mu H_x = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z},$$

(4)

$$\frac{\partial H_x}{\partial y} = 0,$$

(5)

$$\frac{\partial H_y}{\partial z} = 0.$$