MATHEMATICAL MODELING

CORRELATION ANALYSIS OF NONSTATIONARY PROCESSES USING PRIOR INFORMATION

V. Ya. Galkin and V. A. Karpenko

The paper examines the applicability of correlation analysis to identification of some classes of nonstationary processes. Consistency conditions are obtained for the time-dependent 1st order autoregressive (AR) process and for the polynomial AR model of an arbitrary order. These results are applied for stable AR estimation and for linear prediction on given classes of nonstationary processes.

1. Introduction

The interest in correlation and spectral analysis of time series has increased considerably during the last decade [1]. One of the main problems in time series analysis, which often occurs in various applications, is forecasting the future behavior of the time series and developing appropriate control interventions based on the prediction. For stationary processes, the linear prediction problem is easily solved by the Wiener–Kolmogorov approach [2].

In many applications, however, such as mechanics, cardiac diagnosis, speech processing, acoustics, and others, we are dealing with processes which can be regarded as stationary only within limited time intervals or possibly not at all. This necessitates the development of new methods or modification of existing methods specifically for the analysis of various classes of nonstationary processes.

One of the main questions for the development of such methods is the determination of the consistency conditions ensuring that the analyzed process adequately fits some model. This problem reduces to estimating the homogeneity intervals in which the given model may be viewed as describing the particular process. In most cases, such homogeneity intervals are obtained empirically without any formalized criteria. Some authors [3, 4] have developed formalized algorithms to estimate the lengths of homogeneity intervals for some classes of stochastic or deterministic processes.

Below we obtain consistency conditions for first-order autoregression processes with time-dependent parameters. We then use these conditions to obtain reliable estimates of the autoregression coefficients and solve the linear prediction problem for the given class of processes.

2. Estimating the Lengths of Homogeneity Intervals for Time-Dependent First-Order Autoregression Processes

Consider the autoregression process in the form

\[ X(t) = a(t)X(t-1) + \xi(t), \quad X(0) = 0, \quad t = 1, 2, \ldots, \]  

where \( a(t) \) is the autoregression coefficient, \( \xi(t) \) is a stochastic process such that \( E[\xi(t_1)\xi^*(t_2)] = 0 \) for \( t_1 \neq t_2 \) and \( E|\xi(t)|^2 = \sigma^2(t) \) (the star denotes complex conjugates).
As we have noted above, the lengths of homogeneity intervals can be estimated only using prior information about the characteristics of the process [3]. Let us first obtain the conditions of consistency of \( X(t) \) with the first-order autoregressive model

\[
X_\ast(t) = aX_\ast(t-1) + \xi(t), \quad X_\ast(0) = 0, \quad t = 1, 2, \ldots, \tag{2}
\]

where \( a \) is the autoregression coefficient. For simplicity we assume that the homogeneity interval \( I_h \) is symmetric about zero, \( I_h = [-h/2, h/2] \) (a one-parameter interval). The prior information is given in the form

\[
\epsilon_0^{(1)} \leq |a(t)| \leq \epsilon_0^{(2)}, \quad t \in [-h/2, h/2] \tag{3}
\]

with known \( \epsilon_0^{(1)}, \epsilon_0^{(2)} \), where either \( \epsilon_0^{(2)} \leq 1 - \varepsilon \) and \( \epsilon_0^{(1)} > 0 \) or \( \epsilon_0^{(1)} \geq 1 + \varepsilon, \varepsilon \) is a positive number.

Let us introduce the consistency measure for (1) and (2) in the form

\[
D(t) = \|F_1(t, \lambda) - F_2(t, \lambda)\|_1, \tag{4}
\]

where

\[
F_1(t, \lambda) = \frac{\sigma(t)}{e^{\lambda a(t)} - a(t)}, \quad F_2(t, \lambda) = \frac{\sigma(t)}{e^{\lambda a(t)} - a}, \tag{5}
\]

and the norm \( \| \cdot \|_1 \) for every \( t \) is defined by the equality

\[
\|f\|_1 = \int_{-\pi}^{\pi} |f(\lambda)|^2 d\lambda. \tag{6}
\]

Substituting (5) and (6) in (4), we obtain

\[
D(t) = \sigma^2(t)|a(t) - a|^2 \int_{-\pi}^{\pi} \frac{d\lambda}{\|e^{\lambda a(t)} - a(t)\|^2}. \tag{7}
\]

In what follows we assume for definiteness that \( |a| > 1 \) and \( |a(t)| > 1 \) (the case \( |a| < 1, |a(t)| < 1 \) is considered similarly).

Making the change of variables \( e^{\lambda a} = z \), we rewrite (7) in the form

\[
D(t) = -i\sigma^2(t)|a(t) - a|^2 \oint_{|z| = 1} \frac{dz}{z|z - a(t)||z - a|^2}. \tag{8}
\]

The integrand function has three poles inside the circle \( |z| = 1 \). To take the integral, we use the main theorem of residue theory, which gives

\[
D(t) = 2\pi \frac{\sigma^2(t)|a(t) - a|^2}{|a^\ast(t)a^\ast|} \frac{|aa(t) - z_1z_2|}{|(z_1 - a(t))(z_1 - a)(z_2 - a(t))(z_2 - a)|}, \tag{9}
\]

where

\[
z_1 = 1/a^\ast(t), \quad z_2 = 1/a^\ast. \tag{10}
\]

Finally, using (3), we obtain

\[
|D(t)| \leq |a(t) - a|^2 \mu, \tag{11}
\]

\[
\mu = 2\pi \frac{(1 + \epsilon_0^{(1)})^2 + 1}{(e_0^{(1)})(2 + \epsilon_0^{(1)})} \|\sigma^2(t)\|, \quad \|\sigma^2(t)\| = \max_{-h/2 \leq t \leq h/2} \sigma^2(t). \tag{12}
\]