DIFFRACTION OF A PLANE WAVE ON AN ARRAY OF SEMI-INFINITE DIELECTRIC RODS

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A direct numerical method is applied to investigate the problem of diffraction on an array of semi-infinite dielectric rods. The reflection characteristics are analyzed in a wide frequency range. A detailed description of the computational method and the algorithm is given.

The problem of diffraction of electromagnetic waves on transparent periodic structures often arises in the design of radiating systems with a dielectric coating. In this paper, we consider the diffraction of a plane electromagnetic wave on an infinite periodic structure that consists of cylindrical dielectric rods on a dielectric base.

Let 0, x, y, z be a Cartesian system of coordinates. The dielectric structure lies in the region \(-d_2 < z < 0\). All dielectric rods are right circular cylinders of radius R and length \(d_2\) parallel to the z-axis. In the plane \(z = 0\) we also define an oblique system of coordinates \(\xi, \eta\) as shown in Fig. 1, with an angle \(\varphi\) between the axes.

The rod structure is dual-periodic with periods \(a/sin \varphi\) and \(b\) along the \(\xi\)- and \(\eta\)-axes, respectively. The rods are attached to a dielectric base, which is a plane layer \(0 < z < d_1\). The relative dielectric permittivity is a piecewise-constant complex function \(\varepsilon(x, y, z)\), which equals \(\varepsilon_1\) in the base, \(\varepsilon_2\) in the dielectric rods, and 1 throughout the rest of the space; also Im \(\varepsilon(x, y, z)\) \(> 0\). The relative magnetic permeability is everywhere 1.

As an elementary cell of the periodic dielectric structure in space we take the channel \(\{0 \leq x \leq a, 0 \leq y \leq b, -\infty < z < \infty\}\), which in what follows is called a Floquet channel. The longitudinal section of this channel by the plane \(x = a/2\) is shown in Fig. 2.

In this structure we consider harmonic electromagnetic oscillations with time dependence of the form \(\exp(-i\omega t)\). The oscillations are excited by a plane linearly polarized wave, whose electric field is written in standard form as

\[
E = E^{inc} (\nu \cos \psi + b \sin \psi) e^{ik(x \cos \alpha + y \cos \beta + z \cos \gamma)}. \tag{1}
\]

Here \(k = \omega/c = \omega(\varepsilon_0\mu_0)^{1/2}\) is the wave number, \(\cos \alpha, \cos \beta, \cos \gamma\) are the directional cosines of the wave vector \(ka = k\),

\[
k = ka = k[i_x \cos \alpha + i_y \cos \beta + i_z \cos \gamma],
\nu = [i_x, a]/||[i_x, a]||,
\]

\(E^{inc}\) is the amplitude and \(\psi\) the angle of polarization of the incident wave.

It is required to find the scattered field and the electromagnetic field in the entire space.

Solution of this problem reduces to solving a system of Maxwell's equations with appropriate conditions on the boundaries of the dielectric base and the rods with radiation conditions. The incident wave field satisfies the quasi-periodicity (Floquet) conditions \[1\]

\[
u(\xi + n \frac{d}{\sin \varphi}, \eta + mb, z) = u(\xi, \eta, z)e^{i(\Delta z + \Delta m)}, \tag{2}
\]

where \(u\) is any component of the electric or magnetic field.
By the Floquet theorem, the scattered field and the total field also satisfy these conditions. The solution of the original problem posed in the entire space thus reduces to the determination of the electromagnetic field in the Floquet channel with additional conditions (2) on its boundaries. This problem has been solved by a direct numerical method based on simultaneous application of the projection scheme of the incomplete Galerkin method and the matching method [2].

Partition the Floquet channel into domains $D_j$ ($j = 0, 1, 2, 3$) by the planes $z = -d_2$, $z = 0$, and $z = d_1$, as shown in Fig. 2. Denote by $E_{\text{N}}^{j}$ and $H_{\text{N}}^{j}$ the approximate values of the total electric and magnetic fields in the domains $D_j$ ($j = 0, 1, 2, 3$). Identify the components $E_{t,n}^{j}, H_{t,n}^{j}$ transversal relative to the z-axis and the components $E_{z,n}^{j}, H_{z,n}^{j}$ longitudinal relative to the z-axis.

In the domains $D_j$ ($j = 0, 1, 2, 3$) with a homogeneous dielectric filling, the approximate solution may be written as a superposition of particular solutions of Maxwell's equations (Floquet harmonics) [1, 2]:

$$E_{t,n}^{j} = \sum_{n=1}^{N} (A_{t}^{j} e^{i\Gamma_{n}^{j} z} + B_{t}^{j} e^{-i\Gamma_{n}^{j} z}) e_{t,n}^{j}(x,y),$$

$$H_{t,n}^{j} = \sum_{n=1}^{N} (A_{t}^{j} e^{i\Gamma_{n}^{j} z} - B_{t}^{j} e^{-i\Gamma_{n}^{j} z}) h_{t,n}^{j}(x,y),$$

$$E_{z,n}^{j} = \sum_{n=1}^{N} (A_{z}^{j} e^{i\Gamma_{n}^{j} z} - B_{z}^{j} e^{-i\Gamma_{n}^{j} z}) \mu_{z}^{2} u_{n}(x,y),$$

$$H_{z,n}^{j} = \sum_{n=1}^{N} (A_{z}^{j} e^{i\Gamma_{n}^{j} z} + B_{z}^{j} e^{-i\Gamma_{n}^{j} z}) \mu_{z}^{2} u_{n}(x,y).$$

The terms entering the sums in (3) are the transversal components of eigenwaves of two types in the domains $D_j$: E-type waves and H-type waves. For E-type (H-type) waves, the z-component of the magnetic (electric) field is zero. In the sums marked by a single prime in (4) summation is over the indices $n$ that correspond to E-waves, and in the sums marked by a double prime summation is over the indices $n$ corresponding to H-waves:

$$e_{t,n}^{j}(x,y) = \begin{cases} \frac{i\Gamma_{n}^{j}}{\sqrt{\mu_{n}}} \nabla_{x} u_{n} & \text{E-type} \\ ik[\nabla_{2} u_{n}, i_{x}] & \text{H-type} \end{cases},$$

$$h_{t,n}^{j}(x,y) = \begin{cases} -i e_{j} [\nabla_{2} u_{n}, i_{x}] & \text{E-type} \\ i \Gamma_{n}^{h} \nabla_{x} u_{n} & \text{H-type} \end{cases},$$

$$u_{n}(x,y) = \frac{1}{\mu_{n} \sqrt{ab}} \exp(i \alpha_{n} x + i \beta_{n} y),$$

$$\alpha_{n} = -k \cos \alpha + 2 \pi n'(n) a, \quad \beta_{n} = -k \cos \alpha + 2 \pi n'(n) a,$$

$$\mu_{n} = \sqrt{\alpha_{n}^{2} + \beta_{n}^{2}}, \quad \Gamma_{n}^{h} = \sqrt{\mu_{n}^{2} - \mu_{n}'^{2}},$$

$$\text{Re} \Gamma_{n}^{h} > 0, \quad \text{Re} \Gamma_{n}' > 0, \quad 1 \text{m} \Gamma_{n}^{h} > 0,$$

$$\nabla_{2} u_{n} = i_{x} \frac{\partial u_{n}}{\partial x} + i_{y} \frac{\partial u_{n}}{\partial y}.$$