SIMULATION OF PROCESSES IN AN ELECTRODYNAMIC
MASS ACCELERATOR

I. S. Gushchin

Processes in a pulse plasma accelerator used as a mass accelerator are simulated in the magnetohydro-
dynamic approximation. Numerical calculations elucidate the dynamics of formation of a "plasma piston". Two qualitatively different modes of plasma piston formation are identified, and the acceleration of the body in each mode is determined. Calculations show that the plasma dynamics mainly depends on the mass of the working substance, the dependence of plasma conductivity on the thermodynamic parameters, and the electrode cooling scheme.

Processes in erosive magnetohydrodynamic plasma accelerators have been examined in [1-3] from the point of view of their use as electrical jet-propulsion engines. The present study focuses on simulation of processes in pulse plasma accelerators used as mass accelerators.

Assume that the infinite accelerator channel has a rectangular cross section of area $S = d \times a$, where $d$ is the width of the rail electrode and $a$ is the distance between rails (Fig. 1). The left wall of the accelerator is fixed. The $x$-axis points in the direction of the accelerated body, the $y$-axis is parallel to the direction of electrical current in the plasma, and the $z$-axis points in the direction of the magnetic field.

Plasma motion in the accelerator channel is described by the following equations:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &+ \frac{\partial}{\partial x} (\rho v_x) = 0, \\
\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial x} (p + \frac{B_x^2}{2\mu_0}) &= 0, \\
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) &= \frac{\partial W}{\partial x} + \frac{j^2}{\rho} + \frac{Q}{\rho}, \\
\frac{\partial B_z}{\partial t} + \frac{\partial}{\partial x} (v_x B_z) &= \frac{\partial}{\partial x} (-\frac{1}{\mu_0 \sigma} \frac{\partial B_z}{\partial z}), \\
\frac{\partial B_z}{\partial z} &= -\mu_0 j, \quad j = \sigma E, \quad W = -\lambda \frac{\partial T}{\partial z}, \quad \lambda = \lambda_0 = \text{const}, \\
Q &= -(\alpha w_0 + \alpha w_1 v_e) (T - T_w)/d, \\
p &= (1 + \alpha) \frac{R}{M_0} \rho T, \quad \sigma = cv (1 + \alpha) T + (I/k)(R/M_0) \alpha, \\
\alpha^2 &= c_0 T^{3/2} \exp\left(\frac{1}{kT}\right)/\rho, \quad c_v = 3/2 (R/M_0), \\
c_0 &= 2k(M_0/R)(2\pi m_e k^2/\hbar^2)^{3/2}, \\
\sigma &= \begin{cases} 
\sigma(T), & \text{if } \rho \geq \rho_0, \\
\sigma(T)(\rho/\rho_0)^{\delta}, & \text{if } 0 < \rho < \rho_0
\end{cases}
\end{align*}
\]

\(\sigma(T)\) is a tabular function.

Here we use the following notation: \(\rho\) is the plasma density, \(t\) is time, \(x\) is the Eulerian coordinate, \(v_x\) is the plasma velocity, \(p\) is plasma pressure, \(B_z\) is the magnetic induction, \(\mu_0\) is the magnetic permeability of vacuum, \(\sigma\) is the plasma...
Fig. 1. Electrodynamic mass accelerator: 1) power source, 2) accelerator electrodes, 3) accelerated body.

 conductivity, $\varepsilon$ is the plasma internal energy (unit of mass), $W$ is the heat flux, $\lambda$ is the thermal conductivity of plasma, $T$ is the plasma temperature, $j$ is the current density, $Q$ is the power of heat sources or sinks per unit volume, $\alpha_{W_0}$ and $\alpha_{W_1}$ are the heat-transfer coefficients of the accelerator channel walls, $T_W$ is the rail temperature, $R$ is the universal gas constant, $M_0$ is the molar mass of the gas, $k$ is the Boltzmann constant, $c_V$ is the heat capacity at constant volume of unit mass of the gas, $\alpha$ is the degree of ionization, $I$ is the first ionization potential, $m_e$ is the electron mass, $h$ is Planck’s constant, $E$ is the electric field strength, $\rho_0$, $\beta$, $\lambda_0$ are given constants.

System (1) is augmented with boundary and initial conditions. The conditions on the left boundary have the form

$$v_x(0,t) = 0,$$
$$W(0,t) = 0,$$
$$B_z(0,t) = \mu_0 J(t)/d,$$

where $J(t)$ is the total current in the accelerator circuit. The conditions at the right boundary assume the presence of an accelerated body of mass $M$:

$$v_x(x_{\text{max}}(t),t) = 0,$$
$$W(x_{\text{max}}(t),t) = 0,$$
$$B_z(x_{\text{max}}(t),t) = 0,$$

where $x_{\text{max}}(t)$ is the position of the accelerated body relative to the left wall of the accelerator, $p_0$ is the pressure outside the accelerator.

We assume that the plasma parameters at $t = 0$ and for $0 \leq x \leq x_{\text{max}}(0)$ have the following values:

$$v_x(x,0) = 0,$$
$$B_z(x,0) = 0,$$
$$T(x,0) = T_{\text{in}} = \text{const},$$
$$p(x,0) = p_{\text{in}} = \text{const},$$

where $T_{\text{in}}$ and $\rho_{\text{in}}$ are the initial temperature and density of the gas.

The problem was solved by finite-difference methods.

Calculations show that, depending on the conditions of plasma piston formation, we may observe different motion modes of the plasma and the accelerated body, respectively. Let us consider two most typical modes. Parameters common for both modes had the following values: $a = 1 \text{ cm}$, $d = 1 \text{ cm}$, $S = 1 \text{ cm}^2$, $M_0 = 64 \text{ g/mole}$ (which corresponds to a "copper plasma"), $M = 1 \text{ g}$, $R/M_0 = 130 \text{ J/(kg K)}$, $c_V = 195 \text{ J/(kg K)}$, $\lambda_0 = 10^5$ (dimensionless form), $T_W = 300 \text{ K}$, $I = 7.7 \text{ eV}$, $c_\alpha = 5.123 \cdot 10^{-4} \text{ kg/(m}^3\text{ K}^{3/2})$, $\rho_0 = 1 \text{ kg/m}^3$, $p_0 = 10^5 \text{ Pa}$, $\rho_{\text{in}} = 1 \text{ km/m}^3$, $T_{\text{in}} = 769 \text{ K}$.

If the gas mass (initial or produced through intensive erosion of the electrodes) is comparable with the mass of the accelerated body, "an extended plasma piston" mode is observed. In this computational variant, the gas mass is $M_g = 0.1 \text{ g}$, $x_{\text{max}}(0) = 1 \text{ m}$, $\beta = 2$, $\alpha_{W_0} = 0$, $\alpha_{W_1} = 0$, $J_{\text{max}} = 1 \text{ MA}$, $t_p = 100 \mu\text{sec}$, where $J_{\text{max}}$ is the maximum current through the accelerator and $t_p$ is the time of linear growth of the current from 0 to $J_{\text{max}}$ (for $t \geq t_p$ the current is constant and equals $J_{\text{max}}$).

Calculations show that, with these parameters, the processes in the gas evolve in the following manner. The current in the accelerator electric circuit creates a magnetic field. The gas conductivity is initially high (the gas temperature is low, the degree of ionization is also low), and the magnetic field freely penetrates through the gas from the left boundary, where