II. MATHEMATICAL MODELS IN ELECTRODYNAMICS

A METHOD FOR CALCULATING THE CURRENT DISTRIBUTION IN AN UNSYMMETRIC OSCILLATOR WITH SEVERAL EXCITATION POINTS

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A method is proposed for calculating the current distribution in an unsymmetric oscillator with several excitation points joined by a common distribution line. The radiator and the distribution line are represented by multipoles, which are described by scattering matrices. The elements of the radiator scattering matrix are calculated by the integral equation method. The calculation results are compared with experimental findings.

One of the techniques for the development of USW radio communication is by extending the frequency range of the radio instruments. The use of narrow-band antennas in such radio instruments is an obstacle to exploiting the advantages of wide-band communication, and wide-band USW antennas are needed. A wide-band antenna can be designed using an unsymmetric oscillator with several excitation points [1, 2].

Let us consider the problem of calculating the current distribution in such an oscillator. An unsymmetric oscillator with N excitation points joined by a common distribution line is represented by two multipoles: the distribution line and the radiating system (Fig. 1). Multipole 1 corresponds to the N-input radiating system formed by the elements of the oscillator lateral surface. Multipole 2 corresponds to the distribution line which links the generator connected to the (N + 1)-th line input with the excitation points of the unsymmetric oscillator. We use scattering matrices for joint description of parallel-connected multipoles. The application of conductivity matrices [3] involves certain difficulties, because the elements of the multipole conductivity matrix go to infinity at the resonance frequencies of long-line segments.

The scattering matrix elements are defined as

\[ S_{am}^{1,2} = \left. \frac{b_{n}^{1,2}}{a_{m}^{1,2}} \right|_{a_{m}^{1,2} = 0}, \quad p \neq m, \]  

where \( m, n, p \) are the input indices,

\[ a_{m}^{1,2} = \frac{U_{m}^{+}}{\sqrt{W_{m}}}, \quad b_{n}^{1,2} = \frac{U_{n}^{-}}{\sqrt{W_{n}}}, \]

where \( W_{m} \) and \( W_{n} \) are the impedances of the transmission lines connected to inputs \( m \) and \( n \), \( U_{m}^{+} \) and \( U_{n}^{-} \) are the voltage amplitudes corresponding to the incoming wave in input \( m \) and the outgoing wave in input \( n \).

To find \( S_{am} \), we have to solve the problem of excitation of an unsymmetric oscillator by an external generator with internal impedance \( W_{m} \) connected to the m-th input of the radiating system, given that all other inputs are loaded by impedances equal to those of the corresponding transmission lines (Fig. 2). The solution of this problem reduces to solving a system of nonhomogeneous Maxwell’s equations that satisfy the boundary conditions on an ideally conducting oscillator lateral surface and on a plane, and also the radiation conditions at infinity.

The z axis of the cylindrical coordinate system is directed along the oscillator axis, and the origin is placed in the conducting plane. Mirror-reflecting the unsymmetric oscillator, we write the Gallen equation for the radiator [4]:

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Fig. 1. Schematic partition of an antenna with several excitation points into multipoles.

\[ \int_0^1 I(z') K(z, z') \, dz' = C \cos kz + F(z) , \quad (2) \]

where

\[
K(z, z') = \frac{2}{\pi} \int_0^{2\pi} \left\{ \frac{\exp(-ik_0 R)}{R} - \frac{1}{R} + \frac{\exp(-ik_0 R_0)}{R_0} \right\} + \\
+ \frac{1}{\pi a} \left\{ \ln \left( a \pi + \sqrt{(z-z')^2 + a^2 \pi^2} \right) - \ln |z-z'| \right\} , \quad (3)
\]

\[
F(z) = \frac{i}{W_0} \sum_{n=1}^{N} U_n^2 \sin(z-z_n) \, h(z-z_n) ,
\]

\[
R = \sqrt{(z-z')^2 + 4a^2 \sin^2 \varphi} , \quad \hat{R} = \sqrt{(z-z')^2 + 4a^2 \varphi^2} , \quad R_0 = \sqrt{(z+z')^2 + 4a^2 \sin^2 \varphi} , \quad (4)
\]

\[
U_n^1 = \begin{cases} 
E_m - I(z_m) W_m & n = m \\
- I(z_n) W_n & n \neq m \end{cases} , \quad h(z-z) = \begin{cases} 
0 & z < z_m \\
1 & z \geq z_m \end{cases} .
\]