A PRAGMATIC INTERPRETATION OF
INTUITIONISTIC PROPOSITIONAL LOGIC*

ABSTRACT. We construct an extension \( \mathcal{L}^P \) of the standard language \( \mathcal{L} \) of classical propositional logic by adjoining to the alphabet of \( \mathcal{L} \) a new category of logical-pragmatic signs. The well formed formulas of \( \mathcal{L} \) are called radical formulas (rf) of \( \mathcal{L}^P \); rf preceded by the assertion sign \( \vdash \) constitute elementary assertive formulas of \( \mathcal{L}^P \), which can be connected together by means of the pragmatic connectives \( N, K, A, C, E \), so as to obtain the set of all the assertive formulas (af). Every rf of \( \mathcal{L}^P \) is endowed with a truth value defined classically, and every af is endowed with a justification value, defined in terms of the intuitive notion of proof and depending on the truth values of its radical subformulas. In this framework, we define the notion of pragmatic validity in \( \mathcal{L}^P \) and yield a list of criteria of pragmatic validity which hold under the assumption that only classical metalinguistic procedures of proof be accepted. We translate the classical propositional calculus (CPC) and the intuitionistic propositional calculus (IPC) into the assertive part of \( \mathcal{L}^P \) and show that this translation allows us to interpret Intuitionistic Logic as an axiomatic theory of the constructive proof concept rather than an alternative to Classical Logic. Finally, we show that our framework provides a suitable background for discussing classical problems in the philosophy of logic.

1. INTRODUCTION

We propose a pragmatic interpretation of intuitionistic logic that is based on a translation of an intuitionistic propositional calculus (IPC) and of a classical propositional calculus (CPC) into a formalized pragmatic language \( \mathcal{L}^P \); the latter is an extension of Frege's ideographic language, in which the assertion sign is introduced as a constitutive part in the formulas of the logical calculus.

The purpose of our interpretation is mainly philosophical. Indeed we aim to settle the conflicts between classical and intuitionistic logic, and between the classical (correspondence) and the intuitionistic (verificationist) conceptions of truth and meaning (see Dummett, 1977, 1978, 1979, 1980; Prawitz, 1977, 1980, 1987); this will be done by introducing an integrated perspective which preserves both the globality of logic (in the sense of the global pluralism, which admits the existence of a plurality of mutually compatible logical systems, but not of systems which are mutually incompatible or rivals, see Haack 1978, Chapter 12) and the classical notion of truth as correspondence, which we may consider explicited rigorously by Tarski's semantic theory (see Tarski 1933, 1944). This goal is reached in the present paper by translating CPC and IPC into \( \mathcal{L}^P \). Due to the relevance of the subject, we briefly summarize here the essentials of our treatment.

Let \( \mathcal{L} \) be a standard language of the classical propositional logic. We denote by \( \mathcal{L}^P \) in Section 2 an extension of \( \mathcal{L} \), obtained by adjoining to the logical vocabulary (alphabet) of \( \mathcal{L} \) a new category of logical signs,
that we call logical-pragmatic signs, which contains an assertion sign and pragmatic connectives (Definition 2.1.1). By making use of this extended vocabulary, the formation rules of \( \mathcal{L}^p \) recursively define two kinds of well formed formulas in \( \mathcal{L}^p \): the radical formulas (corresponding to the well formed formulas in \( \mathcal{L} \)) and the assertive formulas. Any assertive formula contains radical formulas as proper subformulas (Definition 2.1.2). Then the semantic rules of \( \mathcal{L}^p \) specify the conditions that must be fulfilled, whenever a semantic interpretation of the radical formulas is given by assigning a (classical) truth value to every radical formula of \( \mathcal{L}^p \) (Definition 2.2.1). Furthermore, the pragmatic rules of \( \mathcal{L}^p \) specify the conditions that must be fulfilled, whenever a pragmatic evaluation of the assertive formulas is given by assigning to every assertive formula of \( \mathcal{L}^p \) a justification value (justified or unjustified); this is defined, as the so-called intuitionistic notion of truth, in terms of the intuitive (informal) notion of proof, the assignment being such that the pragmatic evaluation of an assertive formula of \( \mathcal{L}^p \) depends on the (semantic) assignments of truth values to its radical subformulas (Definition 2.3.1). Then, we define in Section 3 the notion of pragmatic validity in \( \mathcal{L}^p \) by using the semantic and pragmatic rules of \( \mathcal{L}^p \), and provide some (direct or indirect) criteria of validity. These are applied in Section 4 in order to explore the relations among semantic and pragmatic connectives in \( \mathcal{L}^p \). The translations of CPC and IPC in \( \mathcal{L}^p \) are then constructed in Section 5 in such a way that the set of all theorems of CPC bijectively corresponds (as in the original Fregcan system) to the set of all elementary assertive formulas which are pragmatically valid in \( \mathcal{L}^p \), while the set of all theorems of IPC bijectively corresponds to the set of all complex assertive formulas (containing only atomic radical formulas) which are pragmatically valid in \( \mathcal{L}^p \). Finally, we discuss in Section 6 some relevant philosophical aspects of our work.

It is interesting to note that \( \mathcal{L}^p \) formalizes, in particular, the analysis of all sentences in terms of force sign and radical introduced by Frege (1879, 1891, 1893, 1918) and developed by various authors, among which Reichenbach (1947, Section 57) and Stenius (1969). Yet, the Frege–Reichenbach–Stenius (FRS) model applies to elementary assertive formulas only (whose pragmatic interpretation is provided in a merely intuitive way); with this model in mind, Frege proposed his system of classical logic in terms of assertive formulas, but he could not have also given a formulation of the intuitionistic logic compatible with his system of classical logic. Our language \( \mathcal{L}^p \) goes beyond the limits of the FRS model by introducing the pragmatic connectives, which allow the construction of complex assertive formulas (together with the definition of a formal pragmatic interpretation), and permit the translation of intuitionistic logic into \( \mathcal{L}^p \).

It should also be noted that our pragmatic interpretation (translation) differs in two basic aspects from the modal interpretation (translation), proposed by Gödel (1933), McKinsey and Tarski (1948), Fitting (1969), which provides a similar solution of the conflict between intuitionistic and