THEORY OF CREEP OF CYCLICALLY UNSTABLE MATERIALS

Yu. I. Kadashevich and S. P. Pomytkin

The structural model of the medium is used as a basis for describing determining relations of the theory of creep of cyclically unstable materials. The relations are based on the concept of the 'memory' surface proposed by N. Ohno. In the theory equations this surface is introduced on the macrolevel which greatly simplifies calculation equations. The experimental data are compared with the calculations carried out using the proposed theory.

A general theory of hardening was proposed previously in [1, 2]. The determining equations of this theory have the form

\[
\begin{align*}
\tau \frac{d\varepsilon_{ij}^p}{d\lambda} &= \sigma_{ij} - b \varepsilon_{ij}^p; \quad \varepsilon_{ij} = \frac{\sigma_{ij}}{2G} + \varepsilon_{ij}^p; \\
\tau &= \tau_0 K(\langle \lambda \rangle, \langle \varepsilon_{ij} \rangle); \quad \langle \varepsilon_{ij}^p \rangle = \int_0^\infty \varepsilon_{ij}^p d\Phi(\tau_0); \\
d\lambda &= (det_{ij}^p)^{1/2}; \quad \lambda = d\lambda/dt; \\
\sigma_{ij} &= \langle \sigma_{ij} \rangle \quad (\text{or } \varepsilon_{ij} = \langle \varepsilon_{ij} \rangle),
\end{align*}
\]

where \(\varepsilon_{ij}\) is the deviator of the tensor of local strains; \(\varepsilon_{ij}^p\) is the deviator of tensor of local inelastic strains; \(\tau\) is the local yield limit of the material; \(\langle \rangle\) is the sign of the averaging operations; \(\Phi(\tau_0)\) is the integral function of distribution of the yield limit of the material in instantaneous loading.

Analysis of these relations shows that they describe, in contrast to the classic theory of hardening, the recovery effect in complete removal of the load. However, experimental data indicate that the detected softening effect [3, 4] requires correction of the theory.

N. Ohno proposed [5] in addition to the loading surface to introduce into the theory of plastic yielding also the internal surface of plasticity (in the space of plastic strains) which can be referred to as the 'memory' surface. The variation of the parameters of this surface makes it possible to separate clearly monotonic and cyclic loading.

It was shown [6] that there is a close relationship between the theory with the memory surfaces and multisurface theories of plasticity of a special type [7, 8]. In the same work [6] it is recommended in describing inelastic deformation by means of a structural model of the medium to disregard the local memory surface and limit considerations to introducing a single macroscopic surface memory. The relations of the theory are greatly simplified mainly by means of the average nature of the parameters.

We shall apply these considerations [5, 6] to the generalized theory of hardening (1). We examine the following group of determining relations:

\[
\begin{align*}
\tau(\lambda_i) \frac{d\varepsilon_{ij}^p}{d\lambda_i} &= \langle \varepsilon_{ij}^p \rangle; \quad d\lambda_i = (det_{ij}^p)^{1/2}; \quad \tau \frac{d\varepsilon_{ij}^p}{d\lambda} &= \sigma_{ij} - b \varepsilon_{ij}^p; \\
\tau &= \tau_0 K(\langle \lambda \rangle, \tau_i); \quad \langle \sigma_{ij} \rangle = \sigma_{ij}; \quad \langle \varepsilon_{ij}^p \rangle = \int_0^\infty \varepsilon_{ij}^p d\Phi(\tau_0);
\end{align*}
\]

The first equation of this group holds for the macroscopic memory surface. In addition, it is assumed that the behavior of this surface does not depend on the rate of inelastic deformation. (If necessary, it is possible to analyze the behavior of the memory surface by means of the dependence of its parameters on \( \langle \lambda' \rangle \).)

To use Eq. (2) in specific calculations for the function \( K(\langle \lambda' \rangle, \tau_1) \) and \( \Phi(\tau_0) \) we accepted the approximations of the following type

\[
K(\langle \lambda' \rangle, \tau_1) = \langle \lambda' \rangle^2 \tau_1^n ;
\]

\[
\Phi(\tau_0) = \Lambda \tau_0^2 .
\]

The parameters of the model were determined from experiments in the conditions of uniaxial creep assuming that the classic theory of hardening \[9\] holds in the following form

\[
\langle \varepsilon^P \rangle = B \langle \sigma \rangle \langle \varepsilon^P \rangle^l , \quad (3)
\]

and also from creep experiments under inverse loading conditions:

\[
\sigma = \sigma_0 , \quad 0 \leq t \leq t_0 ; \quad \sigma = -\sigma_0 , \quad t \geq t_0 .
\]

In processing the experimental data \[3, 4\] for stainless steel 304 at a temperature of 650°C \( \alpha = 34, \quad a = 0.0835, \quad n = 0.0417, \quad A/b = 1.72 \times 10^{-96}. \)

To indicate the possibilities of the theory we shall examine the results of tests obtained under complex cyclic loading (tension with torsion) under creep conditions \[4\]. Figure 1 shows the history of loading and Fig. 2 the experimental and theoretical data obtained from the generalized theory of hardening (1) and (2). The graphs indicate that the generalized theory of hardening describes satisfactorily the behavior of the material from the qualitative viewpoint but as regards quantitative descriptions it is not in agreement with the experimental data. The curves constructed using Eq. (2) indicate that these ratios satisfy the experimental data also quantitatively.

In addition to this variant of theory the authors also examined a more complicated form of the memory surface. It was assumed that there are two memory surfaces. The determining relations have the form

\[
\begin{align*}
\tau_1(\lambda_1) \frac{d\varepsilon^P_{ij}(1)}{d\lambda_1} &= \langle \varepsilon^P_{ij} \rangle - m_i \varepsilon^P_{ij} - n_i \varepsilon^P(2), \\
\tau_2(\lambda_2) \frac{d\varepsilon^P_{ij}(2)}{d\lambda_2} &= \langle \varepsilon^P_{ij} \rangle - m_2 \varepsilon^P_{ij} - n_2 \varepsilon^P(2), \\
\tau_1 &= m_1 \ldots m_1 = m_2 = m, \\
\tau_2 &= m_2 \ldots n_1 = m_2 = 2m, \\
r &= r_0 K(\langle \lambda' \rangle) + q(\tau_1, \tau_2, \langle \lambda' \rangle).
\end{align*}
\]

These relations for the problem as shown in Fig. 1 also gave completely satisfactory results. Specially formulated experiments should show the type of loading in which more complex relations (4) should be used instead of (2). The results will be published.