A probabilistic model is proposed for the long-term strength of polymer materials containing initial cracks of different length. The model is used to determine the probability density of the longevity values and the logarithm of longevity; they are used to explain the experimentally observed spread in the values of the longevity and the form of the empirical probability density for the logarithm of the longevity.

In experimental investigations of polymer longevity it is observed that there is a significant spread (up to six to eight orders of magnitude) in the values of the longevity \([1-3]\), as well as citations in [4]). This large spread makes it very difficult to predict the long-term strength of polymers (Fig. 1, [5]). An obvious way out of this situation is to determine, experimentally or theoretically, the probability distribution of the values of the longevity \(\tau\) or the logarithm of the longevity \(\log_2\tau\), since this function can be used to predict any value of the longevity and establish any numerical characteristics of this random quantity.

Figure 2 displays the probability distribution \(p(\log_2\tau)\), determined experimentally [5] from the data of Fig. 1, of the logarithm of the longevity of a film of polyethylene terephthalate (PETP). As is evident from the figure, the function \(p(\log_2\tau)\) is nonmonotonic and it can have, depending on the stresses, several maxima, which are determined by the presence of different types of defects in the samples [3-5].

The observed spread in the longevity values, just as the form of the function \(p(\log_2\tau)\), is explained, in our opinion, not only by the stochastic nature of the defect structure of the material, but also by the fact that the development of the most dangerous defect of the stressed sample, resulting in fracture of the sample, is a random process. For this reason, a probabilistic approach must be used in order to explain the kinetics of fracture of materials. Such an approach was proposed previously in a number of works [6-10] (see also citations there). Thus in [6-8] the statistical kinetics of fracture of bodies with cracks was studied, but the size distribution of the cracks was neglected. In [9, 10] the volume fracture of a material with nonuniform structure and initial defects was analyzed, but the distribution of structural defects over degrees of danger was neglected.

In the present paper we propose a probabilistic model of the long-term strength of polymer materials which contain initial cracks of different length. The model is used to determine the probability distribution of longevity values and the logarithm of the longevity.

For simplicity we confine our attention to the case of uniaxial stretching of a sample by constant stress \(\sigma\). We describe cracks present in the material in terms of a single parameter — the length \(l\), measured in units of \(\lambda_\ast\) and characterizing the advance of a crack with a single fluctuation \(l = \bar{l}/\lambda_\ast\), where \(\bar{l}\) is the dimensional length of the crack.

In most cases edge cracks are most dangerous. For this reason, we examine the contribution of only such cracks to the longevity of the material. We designate the set of possible lengths of edge cracks in the sample by \(\{l_i\}\), where \(i = 1, 2, \ldots, n\); \(n\) is the possible number of edge cracks of different length; and \(l_i < l_{i+1}\). Here the term "crack" refers to a crack of arbitrary length.

In view of the stochastic nature of the kinetics of fracture of bodies under a load, the longevity \(\tau\) of such bodies is a random continuous quantity, whose values fall in the range \((0, \infty)\). In order to describe this quantity it is sufficient to know the function \(\varphi(t)\) — the probability density that the longevity (i.e., the time from application of the load to the sample up to fracture of the sample) will fall into the time interval \(t\) to \(t + dt\). In order to find this function it is necessary to know how the material fractures within the framework of the thermal-fluctuation mechanism, and this, in turn, is determined by the

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structure of the material and the external conditions: besides mechanical, also temperature, diffusion, and other fields acting on the material. We confine our attention to fracture of material in an inactive medium at constant temperature $T$.

The probability density $\varphi(t)$ depends on the probability that the surface layer of the sample contains initial cracks of different length and the probability density that these cracks reach the critical length $l_c$, corresponding to the onset of the thermal stage of crack growth, whose contribution to longevity we shall assume is small.

We define by $P(l, j, S)$ the probability that there exist $j$ initial edge cracks of length $l_i$ (cracks of the $i$-th type) in the surface layer of a sample with surface area $S$. We denote the average concentration of cracks of type $i$ in the surface layer as $\bar{n}_i$. Then the average surface area of the sample per edge crack of $i$-th type is $\bar{s}_i = 1/\bar{n}_i$. The probability that a crack of the $i$-th type is present in a small area $dS$ of the surface of the sample is $\bar{n}_i dS = dS/\bar{s}_i$. Then the probability that an area $S + dS$ will contain $j$ cracks of the $i$-th type is

$$P(l, j, S + dS) = P(l, j, S) \left(1 - \frac{\Delta S}{\bar{s}_i}\right) + P(l, j - 1, S) \frac{\Delta S}{\bar{s}_i}.$$  

Dividing both sides of Eq. (1) by $\Delta S$ and passing to the limit $\Delta S \to 0$ we obtain

$$\frac{dP(l, j, S)}{dS} = \bar{n}_i \left( P(l, j - 1, S) - P(l, j, S) \right).$$  

For $j = 0$ we write the equation

$$\frac{dP(l, 0, S)}{dS} = -\bar{n}_i P(l, 0, S).$$