DETERMINATION OF WORK DONE AND POWER EXPENDED IN FABRICATION OF PARTS BY PNEUMATIC VACUUM FORMING

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In recent years the annual average growth rate in the volume of materials processed by the pneumatic vacuum forming method has been increasing considerably. The development of the pneumatic vacuum forming method has been aided to an appreciable extent by the increase in the production of, and drop in the cost of, thermoplastic laminate and sheet materials such as celluloid, polymethyl methacrylates, polystyrene and its copolymers, polyvinyl chloride laminates, high-density and low-density polyethylene, polypropylene, polycarbazones, polyethylene terephthalate, etc.

Work devoted to methods for calculating the amount of work done and the power expended in forming some specific part is very much to the point at the present state of the art. The currently existing method of calculations [1] is not generally applicable, since it cannot be used to calculate the work done in pneumatic forming, and does not allow for consideration of the physicomechanical features of the material to be formed.

In determining the amount of work done in the process of forming a part, we have to take into account the fact that, in the case of large strains, i.e., when the amount of change brought about in an element is quite large compared to the size of that element itself, determination of strain from the increment in the length of elements divided by the initial lengths of the unstrained elements in the case of tensile loading yields results which are somewhat too high. The strain should be determined, on that account, from the increment in the lengths of the elements divided by the variable lengths (taken over the time required to complete the deformation process) of the elements [2]:

$$\varepsilon = \frac{1}{l_0} \int \frac{dl}{l} = \ln (1 + \varepsilon),$$

where $\varepsilon$ is the deformation of the material comprising the part; $l$ is the length of the element; $l_0$ is the initial length of the element. Moreover, when the amount of strain is determined by the first method, the sum of the strains in two successive deformation processes will not add up to the total strain from those two successive processes, and the resulting discrepancy will become larger as the strain is increased.

In the case of materials with viscoelastic properties, the specific work done in the forming process can be found from the formula

$$U = \frac{\tau_1}{2} \sigma_1 d \bar{z}_1 + \frac{\tau_2}{2} \sigma_2 d \bar{z}_2 + \frac{\tau_3}{2} \sigma_3 d \bar{z}_3,$$

where $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ are the respective strains in the material of the part in the meridional, equatorial, and radial directions; $\sigma_1$, $\sigma_2$, $\sigma_3$ are the respective stresses in the formed part in the meridional, equatorial, and radial directions; $\varepsilon_K$ is the final logarithmic strain brought about during the forming process.

As an example, we consider determination of the amount of work done in forming a conventional cylindrical sleeve by the straight vacuum forming method. Since we have $\sigma_1 = \sigma_2$, $\sigma_3 = 0$, and $\varepsilon_1 = \varepsilon_2$ when axisymmetric parts are formed by the straight vacuum forming method [3], Eq. (1) then becomes

$$U = \frac{\tau_1}{2} 2 \pi d \bar{z}.$$
The stresses generated in the part at the time of forming can be found from the formula [3]:

\[
\sigma = \frac{pR \exp \left( \frac{K_o h}{R} \right)}{K_y \delta_t},
\]

where \( p \) is the operating pressure; \( R \) is the radius of the part formed; \( K_o \) is the cooling factor; \( h \) is the height coordinate of the cylindrical part; \( K_y \) is the shrinkage factor; \( \delta_t \) is the thickness of the thermoplastic blank.

We determine the specific work done in shaping the walls of a cylindrically shaped part made of viscoelastic material as:

\[
U_{\text{wall}} = 2 \pi \frac{pR}{K_y \delta_t} \exp \left( \frac{K_o h}{R} \right) \left[ 1 + \left( \frac{\exp \frac{K_o H}{0.5 K_y}}{1 - \frac{1}{2 \mu K_o}} \right) \ln \left( \frac{1 + \frac{1}{2 \mu K_o}}{1 - \frac{1}{2 \mu K_o}} \right) \right],
\]

where \( \tilde{e}_k = \ln \left( 1 + \left( \frac{\exp \frac{K_o H}{0.5 K_y}}{1 - \frac{1}{2 \mu K_o}} \right) \right) \); \( H \) is the total height of the cylindrical part; \( \mu \) is Poisson's ratio.

After integrating, we get

\[
U_{\text{wall}} = \frac{2pR}{K_y \delta_t} \exp \left( \frac{K_o h}{R} \right) \ln \left[ 1 + \left( \frac{\exp \frac{K_o H}{0.5 K_y}}{1 - \frac{1}{2 \mu K_o}} \right) \right].
\]

The total work \( A_{\text{wall}} \) done in forming the walls is obtained from the equation

\[
A_{\text{wall}} = \int_0^H U_{\text{wall}} \, dV_{\text{wall}},
\]

where \( dV_{\text{wall}} = 2\pi \delta(h) dh \) is an elemental change in the volume of the walls of the part. The function \( \delta(h) \) is found [4] from the formula

\[
\delta(h) = 0.5 K_y \delta_t \exp \left( -\frac{K_o h}{R} \right).
\]

And accordingly,

\[
dV_{\text{wall}} = \frac{\pi R K_y \delta_t}{\exp \frac{K_o h}{R}} \, dh.
\]

Substituting Eqs. (2) and (4) into Eq. (3), and changing the integration limit, we then determine the total amount of work done in shaping the walls of the cylindrical part:

\[
A_{\text{wall}} = 2 \pi R^2 H \ln \left[ 1 + \left( \frac{\exp \frac{K_o H}{0.5 K_y}}{1 - \frac{1}{2 \mu K_o}} \right) \right].
\]

Similarly, we find the total work done in shaping the bottom of the cylindrical part:

\[
A_b = \pi R^2 \ln \left[ 1 + \left( \frac{\exp \frac{K_o H}{0.5 K_y}}{1 - \frac{1}{2 \mu K_o}} \right) \right].
\]

The total work done in forming the entire part is calculated as the sum of the contributing amounts of work done in shaping the walls and bottom of the part separately:

\[
A = p \pi R^2 (2H + R) \ln \left[ 1 + \left( \frac{\exp \frac{K_o H}{0.5 K_y}}{1 - \frac{1}{2 \mu K_o}} \right) \right].
\]